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# Using Geometry Sketchers and CAD Tools for Mechanism Synthesis

Isabel Prause
Department of Mechanism Theory and Dynamics of Machines, RWTH Aachen University, Aachen, Germany
e-mail: prause@igm.rwth-aachen.de

Jean-Christophe Fauroux
Clermont University, French Institute for Advanced Mechanics (IFMA), Institut Pascal, UMR 6602 UBP/CNRS/IFMA, Clermont-Ferrand, France

Mathias Hüsing
Department of Mechanism Theory and Dynamics of Machines, RWTH Aachen University, Aachen, Germany

Burkhard Corves
Department of Mechanism Theory and Dynamics of Machines, RWTH Aachen University, Aachen, Germany

**Abstract:** Geometry sketchers and CAD Tools can provide a highly valuable help for machine design and industry in the context of mechanism synthesis. They can be used at an early stage of mechanism development when the design is specified in a preliminary way. In order to get a deeper insight in options and features of software tools for the dimensional synthesis, this paper compares the possibilities of two geometry sketchers (Cinderella© and GeoGebra©) and two CAD systems (Inventor Professional©, CATIA V5©). Different synthesis procedures are presented and described in algorithmic form. Necessary functionalities that should be provided by the software tools are derived. Different synthesis examples are shown: a windscreen wiper mechanism and a spherical four-bar linkage.

**Keywords:** Mechanism synthesis, Geometry sketching software, CAD software

## 1 Introduction

The mechanism design process (Fig. 1) can be divided into several steps. First of all, the designer formulates the requirements on the motion task - or future mechanism - such as boundary conditions, specific requirements and the desired actuation concept based on a given problem. Then, the designer searches suitable mechanisms that can fulfill the desired motion task. This step can be done by using mechanism databases or systematic synthesis approaches. To narrow down the number of possible mechanisms, an evaluation and selection step is performed. This results in the best solution that will be pursued for the dimensional synthesis. Here, the task is to find the appropriate dimensions for the mechanism so that the kinematical and dynamical boundary conditions are not violated. Normally, dimensional synthesis is followed by an analysis of the mechanism. However, several iterations are possible within the design process. For example, it can be necessary to go back to the dimensional synthesis if analysis results are not satisfactory. Even from any level *n* it is possible to go back to one or several levels.

Nowadays, classical dimensional synthesis procedures e.g. position synthesis, dead-center position synthesis, etc. (described amongst others in VDI guidelines 2123 [1], 2124 [2], 2125 [3], 2126 [4] and 2130 [5]) and/or analysis methods are less often performed on paper because paper work is generally inaccurate, confusing, cumbersome and static.

Moreover, efficient software tools are available on the market, which can be a great help during the early phases of mechanism development, especially during the conceptual design. Those software tools can be divided into four groups:

- Interactive geometry software (e.g. GeoGebra© [6], Cinderella© [7])
- Specially developed software for mechanism design and analysis (e.g. SAM© [8], Genesys© [9], GECKO© [10], KissSoft© [11] or tools presented in [12, 13])
- CAD systems (e.g. CATIA V5© [14], Autodesk Inventor Professional© [15], PTC Creo© [16], Solid Works© [17])
- Multibody simulation tools (e.g. MSC Adams© [18], LMS Virtual.Lab Motion [19])

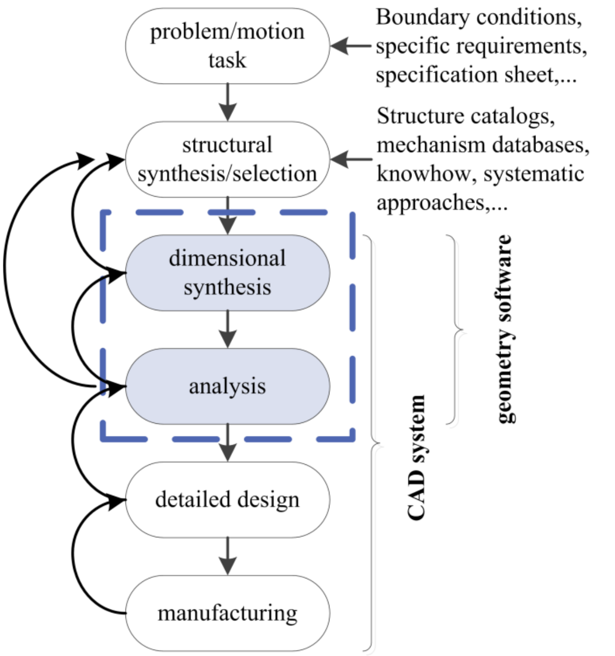


**Fig. 1. Typical design process for a mechanism.**

Interactive Geometry Software (IGS) permits to sketch parametrized geometry and to modify it interactively. Initially developed for educational purposes, IGS can provide a highly valuable help for machine design and industry. It can be used at an early stage of mechanism development when the design is specified in a preliminary way, mostly as a mechanism sketch with incomplete data that can serve as a skeleton for the future mechanism. IGS offers a more powerful access to geometry than the simple use of paper, compass and ruler, because of high construction precision, a posteriori edition capacity and parametrizing. Not only these classical geometry tools are provided, but also additional tools such as spreadsheet, algebraic representation or even symbolic calculus. IGS allows representing a mechanism by its simplified model, often called “skeleton” where the links are represented by lines and the joints are reduced to their center and/or motion axis [35]. Skeleton modeling allows dimensional synthesis of certain links long before the first implementation in a CAD system. Moreover, it is ideal for iterative design. Thereby, a broad analysis of quickly generated mechanism is possible.

Especially in the field of teaching, the use of software tools plays a major role. The aim is that students are familiar with geometrical design principles and easily understand the motion behavior of different kinds of mechanism. Regarding graphical synthesis methods, the students can well understand the effects caused by changes of kinematic parameters on the mechanism behavior.

Hence, IGS is not only used at the Department of Mechanism Theory and Dynamics of Machines for education purposes as explained in [20, 21], but also in other departments. For example, [13] presents the structure of a mechanism toolkit that allows students to write simple programs to solve complicated planar mechanism problems. Education purposes were at the origin of the program MECAN4 [12] as well. It allows simulation of many four-bar linkages and slider-crank mechanisms and includes dimensional synthesis and analysis. The geometry software Cinderella© is both used at IGM and the Faculty of Mechanical Engineering in Niš to teach the principles of mechanism theory [22].

Nevertheless, results of a critical review on using IGS in classes by Gawlick [23] show that IGS should not be considered as a self-operating option in education because courses have to be defined and prepared carefully. Gawlick [23] concludes that new teaching sequences have to be developed so that the use of IGS is based on fundamental geometry knowledge that students acquire beforehand.

But IGS can also be used for research and industrial applications as valuable tools for mechanisms synthesis and analysis.

In order to get a deeper insight in using software tools for the dimensional synthesis and analysis, this paper compares the possibilities of two IGS (Cinderella© and GeoGebra©) and two CAD systems (Autodesk Inventor Professional©, CATIA V5©), especially in the context of educational and engineering purposes. Furthermore, strengths and weaknesses of the mentioned software tools are illustrated.

First, in section 2, relevant synthesis procedures are presented and necessary software requirements are derived. A dead-center position synthesis and a three position synthesis are performed within the mentioned programs using the example of a windscreen wiper. Furthermore, spherical mechanism synthesis is highlighted within the IGS GeoGebra©. The section concludes with a comparison concerning different functionalities. In conclusion, the capacities and limitations of both IGS and CAD systems are summarized with regard to synthesis of mechanisms.

## 2 Comparison concerning synthesis methods

### 2.1 Synthesis procedures

In order to analyze relevant functional requirements for the considered programs, relevant **synthesis procedures** for crank mechanisms are presented [24–26]. Those contain:

- Three position synthesis
- Substitution mechanisms (ROBERTS-CHEBYSHEV-theorem)
- Dead-center position synthesis

For the synthesis procedures the following general definitions are introduced for a four-bar-mechanism:

- $A_0$: rotating point of the crank (frame joint)
- $B_0$: rotating point of the rocker (frame joint)
- A: coupling joint, connects crank with coupler
- B: coupling joint, connects coupler with rocker

#### *2.1.1 Three position synthesis*

The three position synthesis allows the dimensional synthesis of mechanisms that pass through three given poses. The principle is based on the midpoint search for three predetermined positions on a circular path. An example for position synthesis should be given (synthesis of four-bar linkage $A_0ABB_0$). Here, the two frame joints $A_0$ and $B_0$ are known (Fig. 2). Furthermore, three poses (as reference frames) for the coupler link are provided. The objective is to find the positions of the coupling joints A and B in pose 1. The sought joint A connects two links, the crank and the coupler. The position of the crank is unknown, but for the coupler three relative poses (position $O_i$ and orientation $\varphi_i$ of the three reference frames, Fig. 2) are known. Therefore, the coupler is treated as reference in this case. On the crank, the absolute position of joint $A_0$ is known. It is the same for all positions, because $A_0$ is a frame joint. The following definitions are introduced:

- $A_i$ (i=1...3) = the three positions of A
- $B_i$ (i=1...3) = the three positions of B
- $R_i$ (i=1...3) = orthogonal directed reference frame of origin $O_i$ defining the pose i of the coupler AB, first axis $\xi_i$ and second axis $\eta_i$
- $P_j^i$: point that has a relative position in frame *i* identical to the relative position of P in frame *j*

To design the mechanism in pose 1, the subsequent synthesis steps are performed (Fig. 3):

- $A_{0,2}^1$: **transferred_point**(point $A_0$, from frame 2, to frame 1)
- $A_{0,3}^1$: **transferred_point**(point $A_0$, from frame 3, to frame 1)

It is assumed that $A_0$ rotates around A (frame change), i.e. A is the circle center of the circle through $A_{0,2}^1$, $A_{0,3}^1$ and $A_0$:

- $A_1$: **intersection**(**right_bisector**( $A_{0,2}^1$ , $A_{0,3}^1$ ), **right_bisector**($A_{0,2}^1$, $A_0$))

The synthesis procedure is the same to find the position of joint B.

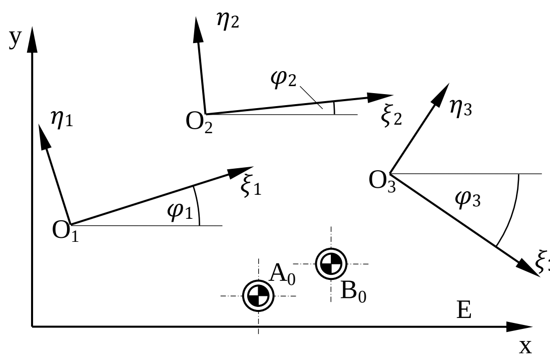


**Fig. 2. Three position synthesis: problem – start sheet.**

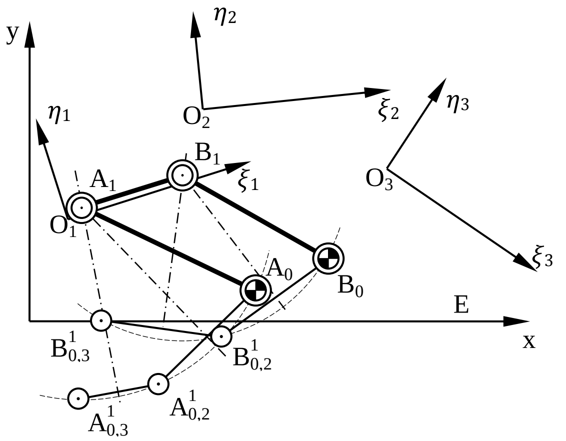


**Fig. 3. Three position synthesis: sketching procedure.**

### *2.1.2 ROBERTS-CHEBYSHEV-theorem*

By using the ROBERTS-CHEBYSHEV-theorem two mechanisms are designed which can create the same coupler curve of a coupler point K as of the initial mechanism (Fig. **4**). Following additional definitions are necessary to describe the ROBERTS-CHEBYSHEV synthesis procedure:

- K: point on the coupler link
- ABK: triangular coupler link
- $\kappa = \sphericalangle BAK$
- $\lambda = \sphericalangle KBA$
- $k = \overline{AK}$
- $l = \overline{BK}$

The synthesis procedure is as follows (for the first mechanism (*) - $A_0A^*B^*B_0^*$, Fig. 4, drawn in dotted lines):

- Point $A^*$
  = so that $A_0A^*KA$ is a **parallelogram**
  = **intersection**(line **parallel**(line $A_0A$, point K), line **parallel**(line AK, point $A_0$))
  or = **intersection**(**circle**(centre K, radius $AA_0$), **circle**(center $A_0$, radius AK))

- Point B*
  = so that triangle (A*B*K) is **homothetic** to triangle (AKB) in this order
  = **intersection**(**circle**(center A*, radius AK·A*K / AB), **circle**(center K, radius BK ·A*K / AB) )
  or = **intersection**(**angular_line**(angle $\kappa$, point A*, A*K), **angular_line**(angle $\lambda$, point K, KA*)

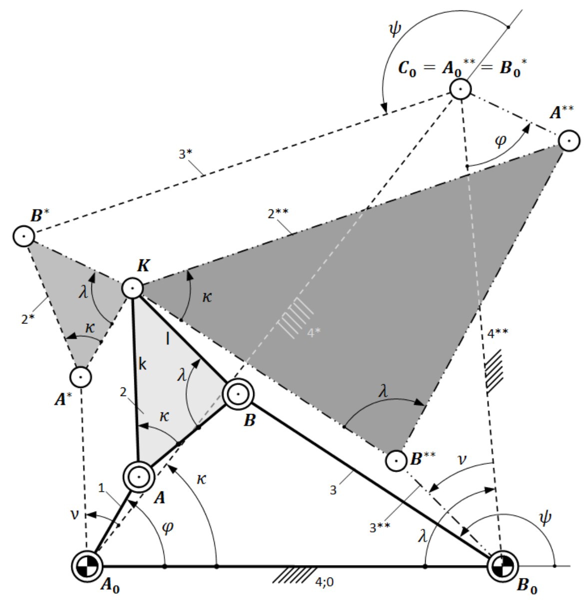

**Fig. 4. ROBERTS-CHEBYSHEV-theorem [26].**

- Point $B_0$*
  = so that triangle ($A_0B_0{}^*B_0$) is **homothetic** to triangle (AKB) in this order
  = **intersection**(**circle**(center $A_0$, radius AK·$A_0B_0$ / AB), **circle**(center $B_0$, radius BK·$A_0B_0$ / AB))
  (or use angles $\kappa$ and $\lambda$)

A similar procedure can be derived for the second mechanism (**) - $A_0$**A**B**$B_0$.

### *2.1.3 Dead-center position synthesis*

If a mechanism is in a dead-center position, the motion of the output link is reversed, whereby the actuated link moves constantly. The dimensions of such a mechanism can be determined by using the dead-center position synthesis procedure. The principle of dead-center position synthesis is based on the theorem of center angle. The objective is to synthesize a four-bar linkage. In general, the swinging angle $\psi$, the rocker length ($l_3$) and the time ratio between forward and backward motion (and hence the crank angle $\varphi_H$) and additionally one other parameter are known (Fig. 5). This parameter can be the crank length ($l_1$), the coupler length ($l_2$) or the eccentricity *e*.

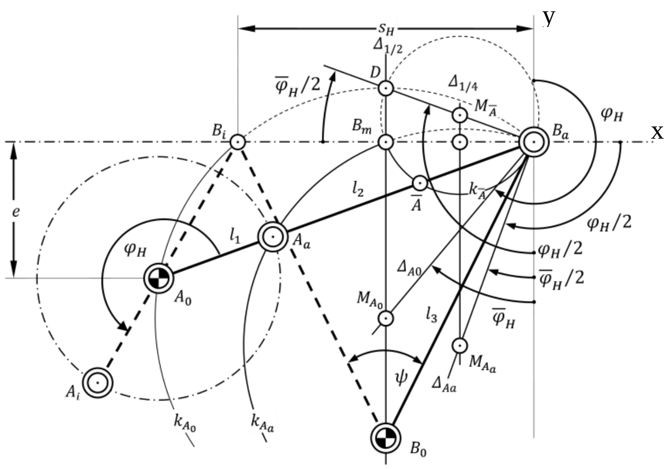


**Fig. 5. Dead-center position synthesis (procedure).**

Two circles, necessary for the synthesis, can be found. The circle $k_{A0}$ which is the locus of all possible positions for $A_0$ and the circle $k_{Aa}$, the locus of all possible positions for $A_a$ (A in the outer dead-center position). To summarize, the following additional definitions are used (referred to Fig. 5):

- $A_i$ : coupling joint A in the inner dead-center position
- $A_a$ : coupling joint A in the outer dead-center position
- $B_i$ : coupling joint B in the inner dead-center position (dead-end of translational stroke)
- $B_a$ : coupling joint B in the outer dead-center position (dead-end of translational stroke)
- $k_{A0}$: **circle** on which $A_0$ is located, center $M_{A0}$
- $k_{Aa}$: **circle** on which $A_a$ is located, center $M_{Aa}$
- $\varphi_H$: $\sphericalangle A_a A_0 A_i$, **angle** centered in $A_0$ and oriented from $A_a$ to $A_i$, $\overline{\varphi_H} = \pi - \varphi_H$
- $\psi$: $\sphericalangle B_a B_0 B_i$, swinging **angle** centered in $B_0$ and oriented from $B_a$ to $B_i$

The following steps have to be performed to find the two circles and hence to get the remaining kinematic dimensions.

- **Preliminary**
  - Construct *x*-axis as the **half-line** starting in $B_a$ directed by $\overrightarrow{B_aB_i}$
  - Construct *y*-axis as the **perpendicular** (point $B_a$, *x*-axis,) such that (*x*,*y*) is direct → *y* = **angular_line** (90°, point $B_a$, *x*-axis)
- **Circle $k_{A0}$**
  - Construct line $(\Delta_{1/2})$ = **right-bisector** of the stroke segment $[B_iB_a]$ = **perpendicular** (point $B_m$ , $(B_iB_a)$)
  - Construct line $(\Delta_{A0})$ = **angular_line** (angle $\overline{\varphi_H}$, point $B_a$, *y*-axis )
  - Construct point $M_{A0}$ = **intersection** $((\Delta_{1/2}), (\Delta_{A0}))$
  - Construct circle $k_{A0}$ = **circle**(center $M_{A0}$, radius $M_{A0}B_a$)
- **Circle $k_{Aa}$**
  - Construct line $(\Delta_{1/4})$ = **right-bisector** of the half-stroke segment $[B_mB_a]$
  - Construct line $(\Delta_{Aa})$ = **angular_line** ($\bar{\varphi}_H/2$ , point $B_a$, *x*-axis )
  - Construct point $M_{Aa}$ = **intersection** $((\Delta_{1/4}), (\Delta_{Aa}))$
  - Construct circle $k_{Aa}$ = **circle**(center $M_{Aa}$, radius $M_{Aa}B_a$)

For example, if the eccentricity (*e*) is given, it can be sketched from the straight line through $B_a$ and $B_i$. The intersection between the circle $k_{A0}$ and line parallel to the *y*-axis, defined by the eccentricity, gives the position of joint $A_0$. Connecting $A_0$ with $B_a$ results in the crank and coupler length.

### 2.1.4 Necessary operations and required tools

Based on the synthesis procedures described previously, necessary operations can be summarized. For the three position synthesis, they are:

- 4 x **circle** (point, radius)
- 3 x **intersection** (circle, circle)
- 2 x **right-bisector** (point, point)

For the *ROBERTS-CHEBYSHEV*-theorem the following operations are necessary to find an alternative mechanism, either at high level:

- 1 x **parallelogram** (point, point, point, point)
- 2 x **homothetic** (triangle, triangle)

or at a lower level:

- 4 x **circle** (point, radius)
- 3 x **intersection** (circle, circle)
- 2 x **parallel** (line, point)

For the dead-center position synthesis the following operations are necessary:

- 2 x **circle** (point, radius)
- 2 x **intersection** (line, line)
- 2 x **right-bisector** (point, line)
- 3 x **angular_line** (angle, point, line)
- 1 x **half-line** (point, vector)

Finally, with regard to the geometric functions, the following tools within a geometry sketcher (IGS respectively) or CAD system are necessary:

- Draw circles/straight lines through given points
- Transfer lengths (compass tool)
- Transfer angles
- Transfer points
- Draw parallel lines
- Draw perpendicular bisectors
- Generate intersection points between circles or lines
- (Generate coupler curves)

## 2.2 Planar mechanism synthesis using different programs

### 2.2.1 Synthesis with IGS

IGS tools available on the market differ in ease of use, graphic user interface, properties, functionalities, etc. To find out which software is the best for mechanism synthesis and analysis purposes, [27] and [28] present a comparison of common IGS used in different fields regarding synthesis and analysis procedures known from mechanism theory.

The results show that GeoGebra© offers many advantages with regard to interface and design methodology compared to other IGS. These include amongst others the easier transfer of angles and the mouse-integrated zoom- and pan-function. Once the angle is measured, it can be parametrized and further used. This reduces the error rate and speeds up the design process.

Furthermore, a tool to directly draw perpendicular bisectors is available. Thus, less construction elements are necessary and the clarity is improved. Corves et al. show in [29] an approach to implement a synthesis and interactive process strategy by using the IGS Cinderella©.

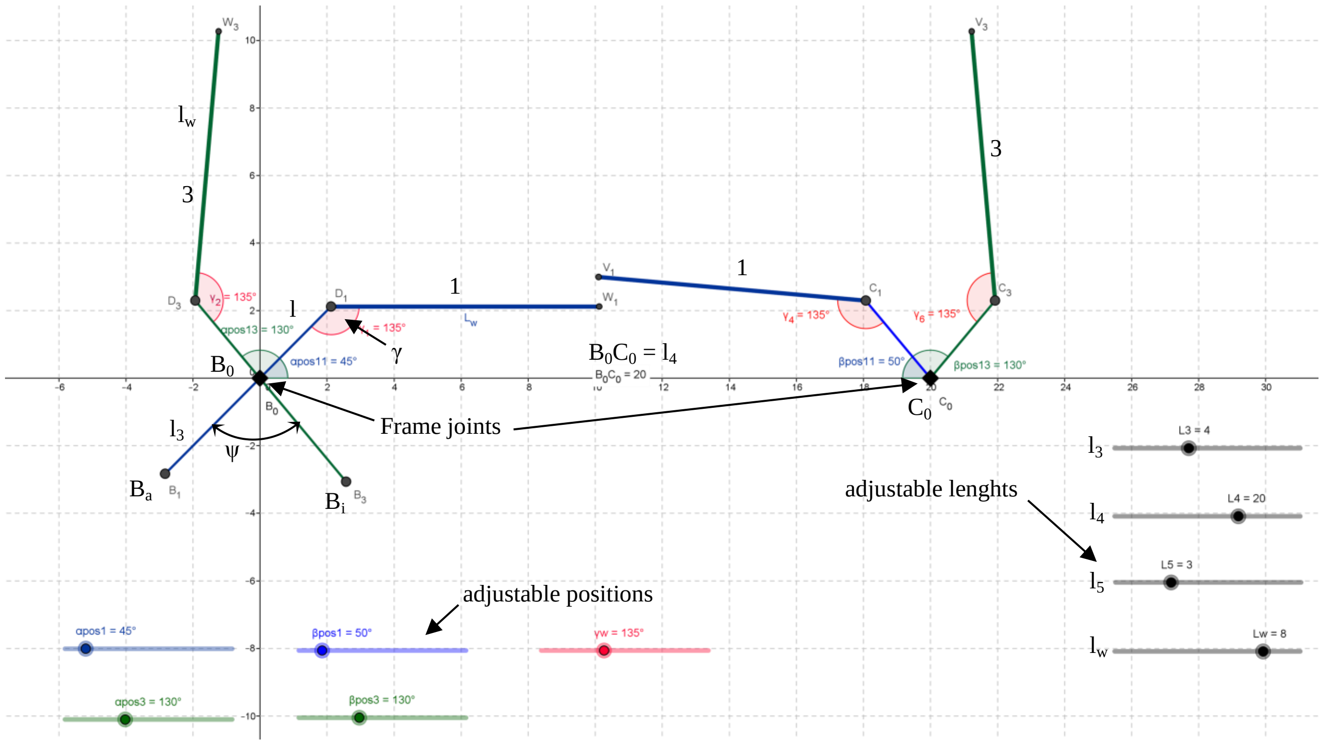


**Fig. 6. Windscreen wiper start file.**

In the first example a convertible roof mechanism is treated, in the second example a roof support is considered which allows straight line guidance. Besides, the application of graphical methods for kinematic dimensioning, a geometry-based power analysis is implemented, so that driving and joint forces are represented in the form of position-dependent force vectors.

As the previous mentioned comparative works [27, 28] showed that GeoGebra exceeds the possibilities of Cinderella, GeoGebra will be used in this paper to develop an interactive worksheet for the dimensional synthesis of a windscreen wiper. Such an interactive worksheet is presented in Fig. 6. It offers the following adjustments:

- Angle positions of the left and right wiper blades – *blue*: position one (1), *green*: position three (3)
- Link lengths (wiper blades, coupler, distance between the two frame joints)
- Attack angle $\gamma$ of the wiper blades
- Position of coupling joint K (Fig. 7) for connection between left and right wiper blade (angle and length)

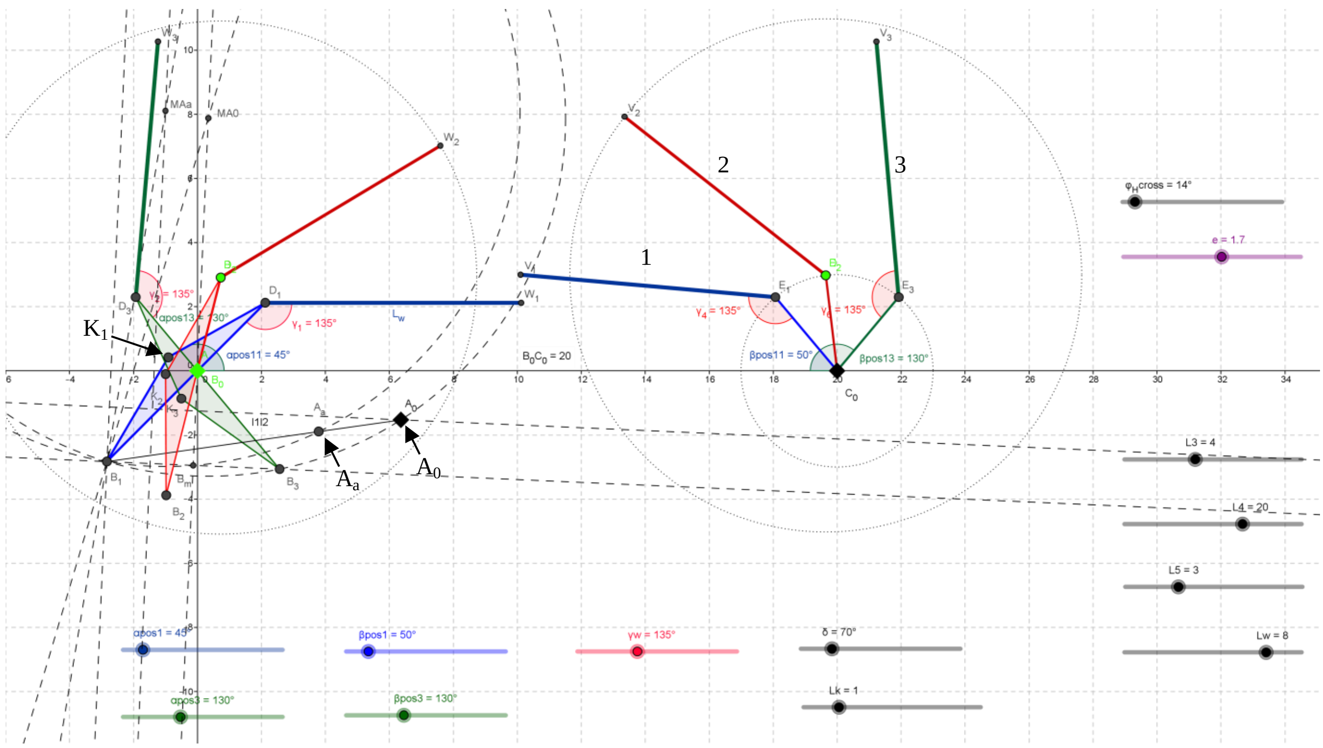


**Fig. 7. Actuation mechanism for left wiper blade.**

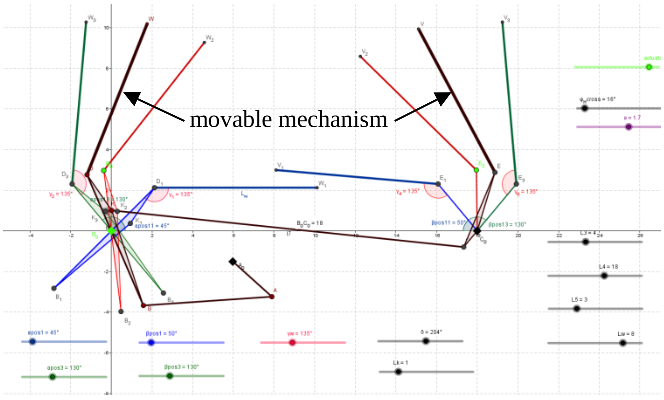


**Fig. 8. Final windscreen wiper mechanism (GeoGebra©).**

First, for this task, the four-bar linkage to actuate the left wiper blade is sought (missing: coupler and crank length, position of crank frame joint $A_0$). The inner and outer dead-center positions, i.e. the swinging angle $\psi$ and the rocker length ($l_3$), the time ratio between forward and backward motion (corresponds to crank angle $\varphi_H$) and the eccentricity $e$ are given in the starting file. Fig. 7 shows the resulting mechanism by applying the *dead-center position synthesis*. In this figure the blades are already sketched in position two, necessary for the further synthesis.

Second, for this task, the two-bar connection between left and right wiper blade is sought. Here, the lengths of both links have to be defined. The problem is solved by applying the *three position synthesis*. From the left blade three poses of the coupling joint K are known, whereas from the right blade three positions are known (Fig. 7). The final movable mechanism is shown in Fig. 8 (*brown*). The same dimensional synthesis can be performed in Cinderella©. As already mentioned, the dimensional synthesis with GeoGebra© is more user-friendly than those with Cinderella©. Additional functionalities and tools in GeoGebra© compared to Cinderella© that facilitate the work are the following:

- Quick transfer of angles (parametrization, “rotation” tool)
- Easy parametrization of the models through sliders (Fig. 6)
- Display of all geometrical objects in algebraic equations (e.g. coordinates, equations)
- Integration of dependencies (e.g. value of eccentricity ($e$))
- Free labeling of construction/geometric elements
- Easy fade-out of non-required geometric elements (amongst other definition of help objects)
- Mouse zoom- and pan-functions

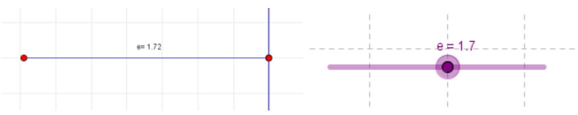


**Fig. 9. Parametrization in Cinderella© (left) and GeoGebra© (right).**

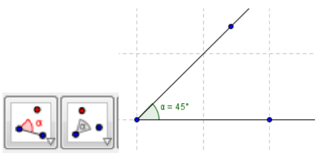

**Fig. 10. Angle transfer in GeoGebra©.**

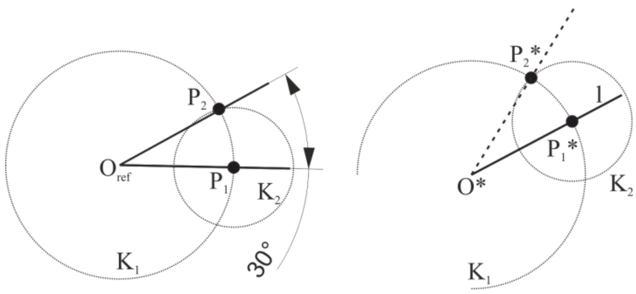


**Fig. 11 Angle transfer in Cinderella©.**

Three functionalities should be described in more detail to understand the differences better. First of all, in GeoGebra© only one operation is used to parametrize parameters (slider tool) (Fig. 9 (right). All properties can be defined in the pop-up window. In Cinderella© no specific tool is available. Starting from a point, a straight line - perpendicular to an axis - is drawn (Fig. 9 (left)). A second point is defined on this line (movable). Then, the line is faded out and the two points are connected by a line segment. Hence, in total of four operations are necessary to define a slider with Cinderella, whereas only one is necessary with GeoGebra©).

Angles are transferred easily in GeoGebra©. There are two options (cf. Fig. 10): Using the angle transfer tool (fixed value) needs two operations. First, a point is rotated around a center and then the second leg is drawn. Second, the rotating tool can rotate a selected line directly around a point.

Angle transfer is more complicated in Cinderella©. The procedure is shown in Fig. 11. First, a circle with fixed radius $K_1$ is sketched around the reference rotating point $O_{ref}$. The intersection of this circle with the legs provides two points $P_1$ and $P_2$ (needed as construction points). Then, the compass tool is used to transfer the circle $K_1$ to the new rotating point O*. The intersection with the leg l gives the new first construction point ($P_1$*). This is the center of circle $K_2$, transferred from the reference figure as well. Intersection of circle $K_1$ and $K_2$ gives the second construction point $P_2$*. A half-line starting from the new rotation point O* through the second construction point $P_2$* defines the second leg (dashed-line). In total this procedure requires eight operations.

As position synthesis is based on midpoint search, perpendicular bisectors are used for classical mechanism synthesis on paper. In GeoGebra© there is a tool that is able to directly draw perpendicular bisectors between two points. If this operation is performed twice, the intersection of both perpendicular bisectors defines the center of the circle through the three points. Here, in total, three operations are required. Cinderella© has no tool for perpendicular bisectors. Therefore, the design procedure is as follows:

- Connecting two points
- Find midpoint between those points
- Draw perpendicular line through this midpoint

The mentioned steps are performed twice. The intersection of both perpendicular lines gives the midpoint. So there are seven operations in total.

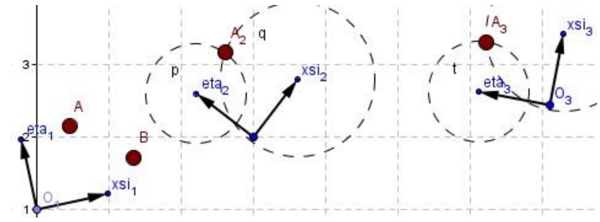


**Fig. 12. Transferring points from one reference frame to other reference frames**

But it is not necessary in IGS to follow the classical approach. In GeoGebra© and Cinderella© a tool is provided to find the circle through three given points and to define the center of that circle. This requires only two operations. Points are transferred using the compass tool twice. This is the same for GeoGebra© and Cinderella©. The procedure (as described in section 2.1.1) is shown in Fig. 12. Instead of drawing circles for the point transfer, reference angles can be used.

*2.2.2 Synthesis with CAD systems*

In order to demonstrate and better visualize the synthesis procedure, the dimensions of the mechanism are scaled. Fig. 9 shows such a mechanism in Autodesk Inventor© with appropriate dimensions. To find appropriate dimensions and design such a mechanism, the mentioned graphical synthesis method can also be implemented in a CAD program. As an example, Lonij et al. [30] show this approach on the basis of a bottle-handling mechanism using the CAD system Autodesk Inventor Professional 2012©. For each synthesis step separate models are created which are later combined to an assembly. By using parametrization, changes in initial parameters can be easily transferred to the dimensions of the final model. In this case, design rules are used for optimization of the mechanism (iLogic feature of Autodesk Inventor©). The focus lies on a limited number of parameters whilst compliance with the predefined requirements is guaranteed. The user can directly observe the influence of parameter changes with respect to performance properties.

Another example for including graphical position synthesis and analysis steps for mechanism development in CAD systems is shown in [31]. The example here is the opening and closing mechanism of a skylight dome.

More generally, Scherer et al. [32] analyze different possibilities to transfer mechanism problems which require graphical synthesis procedures into CAD systems, including Catia V5© and Pro/Engineer Wildfire©. The results show that in general, the integration of graphical synthesis according to VDI guidelines is possible even if design problems due to various CAD functionalities could occur. They conclude that graphical synthesis procedures for mechanism design will still be important in the future. CAD systems may become natural tools for mechanism synthesis, as they are capable to represent 3D multibody assemblies and offer numerous ways to parametrize them. Moreover, further analysis tools are available within CAD software, e.g. installation space/workspace analyses, collision analyses or, as mentioned before, kinetostatic analysis modules.

The aim of the following work is to perform the dead-center position synthesis and the three position synthesis for the windscreen wiper in CAD systems as well. The synthesis procedure is shown below with CATIA V5©.

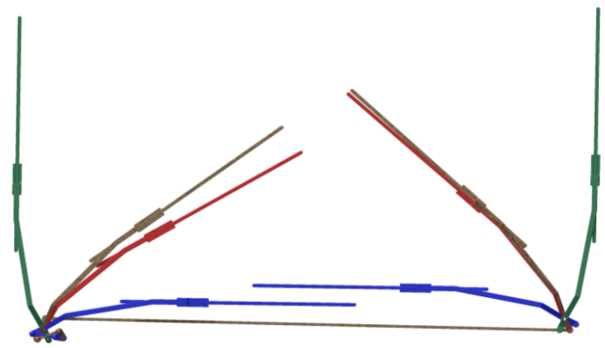

**Fig. 13. Windscreen wiper mechanism with appropriate dimensions (built in Autodesk Inventor©).**

The initial position of the wiper blades is defined by using reference planes in the 3D assembly mode. For this, a new part has to be defined as *skeleton part*. If the orientation angles of those planes are defined as global parameters, they can be changed easily afterwards. Fig. 14 shows the two poses of the left wiper blade.

A new part is inserted for the dead-center position synthesis. Within the sketch mode, the positions of the rocker, points $B_a$ and $B_i$, can be projected in the sketch plane. Then, the dead-center position synthesis can be performed almost like in IGS. By using the snapping tool, perpendicular or parallel axes can be drawn.

Finally, to find crank and coupler lengths, the eccentricity ($e$) (previously defined as global parameter) is sketched from the straight line through $B_a$ and $B_i$. The intersection with the outer circle results in the position of $A_0$. By connecting $A_0$ with $B_a$, the position of A in the outer dead-center position is found.

The lengths of crank ($l_1$) and coupler ($l_2$) are measured and the measured values are kept. The lengths are defined as parameters in the skeleton part and have to be published to be used in other parts. Moreover, the point $A_0$ is defined as an output feature. By doing so, the parts *crank* and *coupler* can be linked to the synthesized lengths (see Fig. 15). An axis through $A_0$ and perpendicular to the wiper plane defines the revolute joint axis of the frame joint $A_0$. For the right side of the windscreen mechanism (Fig. 16), the three positions of the blades are sketched in the *skeleton model* and published (also depending on global parameters). Then, a new part is created for the three position synthesis.

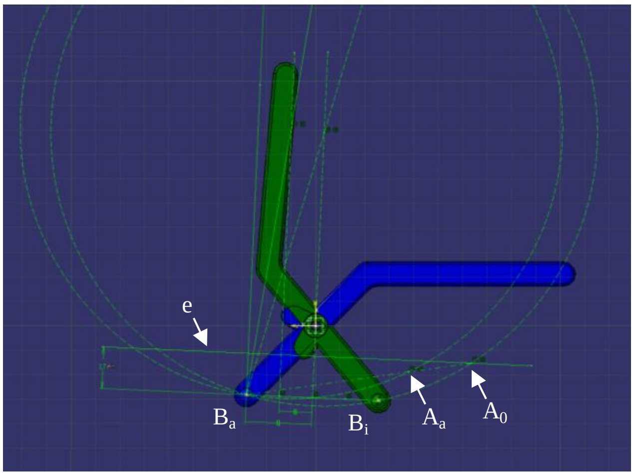


**Fig. 14. Dead-center position synthesis (CATIA© sketch mode).**

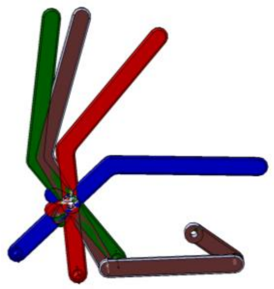

**Fig. 15. Final mechanism for left wiper blade.**

The positions of joint K are projected in the sketch plane and are transferred to the reference position (here position 1, blue wiper). This is done by first measuring the distances between point $K_2$ and $R_2$ and $K_2$ and $C_0$ and then by linking those distances to the triangle $R_1$, $C_0$, $K_2^1$.

In CAD systems, it is not necessary to draw perpendicular bisectors to find the midpoint of the circle passing through the three transferred points, because a tool for finding a circle based on three points is available (in contrast to IGS including the definition of the midpoint). So, the point $C_1$ is defined and the lengths of the two-bar linkage connecting left and right wiper blade are determined. As before, the measured lengths have to be published and re-used (by inter-part reference link) within the respective parts. The movable mechanism can be designed.

The entire dimensional synthesis procedure in CATIA© is not as intuitive as in GeoGebra©. CATIA© is less tolerant concerning mistakes. The designer has to consider the synthesis steps precisely, decide which points have to be published and choose the right references. Otherwise, the mechanism will not be adaptable later.

The entire mechanism can be designed in Autodesk Inventor© using the sketching tool as well. Due to limited space, the synthesis procedure in this software is omitted here. But a proper built windscreen wiper mechanism is shown in Fig. 13. For additional information, in [30] a detailed description of the interactive design of opening and closing mechanisms for skylight domes based on synthesis methods using Autodesk Inventor© is presented.

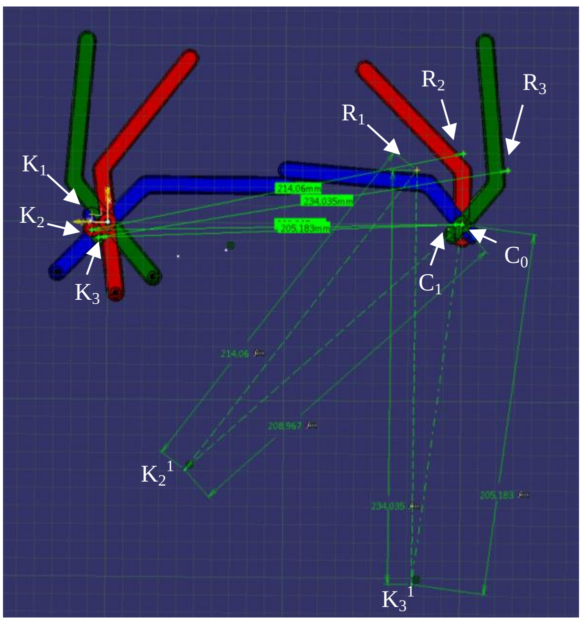


**Fig. 16. Three position synthesis in sketch mode (CATIA©).**

To conclude, compared to Autodesk Inventor©, CATIA© has the following disadvantages with regard to dimensional synthesis procedures:

- Time-consuming publishing of parameters
- Time-consuming definition of output features
- No possibility to define lines with same length without measurement (cf. [32])
- No possibility to create points, straight lines and planes in assembly mode

Defining parameters is similar in CATIA© and Inventor©. But one drawback of CATIA© concerning synthesis is the fact that geometric objects (such as points) have to be published to be globally used. In Inventor© this step is not required.

## 2.3 Mechanism synthesis for spherical mechanisms

Examples for synthesizing spherical or even spatial mechanisms with special developed software can be found in [33] or [34]. In these cases, the software GECKO is used. But spherical mechanism synthesis is also possible by using GeoGebra©. Fig. 17 - Fig. 19 show two examples.

In Fig. 17 a spherical mechanism synthesis is performed. The method for three position synthesis for planar mechanisms can be easily adapted to spherical mechanisms. But in contrast to the procedure described in section 2.1.1., in this example, three poses of the coupler (1-3, joints A and B) are given and the frame joints of crank ($A_0$) and rocker ($B_0$) are sought. This is an example for a spherical tow coupling, described in detail in [34].

Spherical mechanisms have one center point O and the circular trajectories are located on concentric spherical surfaces S centered on that point. To design the mechanism (here to find joint $A_0$), the following synthesis steps are performed:

- Construct circular segment ($A_1$, $A_2$) = **segment** [$A_1$,$A_2$]
- Construct circular segment ($A_2$, $A_3$) = **segment** [$A_2$,$A_3$]
- Construct perpendicular circle ($A_1$, $A_2$) = **right_circle**(center O, equidistant to $A_1$, $A_2$) = $C_1$
- Construct perpendicular circle ($A_2$, $A_3$) = **right_circle**(center O, equidistant to $A_2$, $A_3$) = $C_2$
- Construct point $A_0$: **intersection**($C_1$, $C_2$)

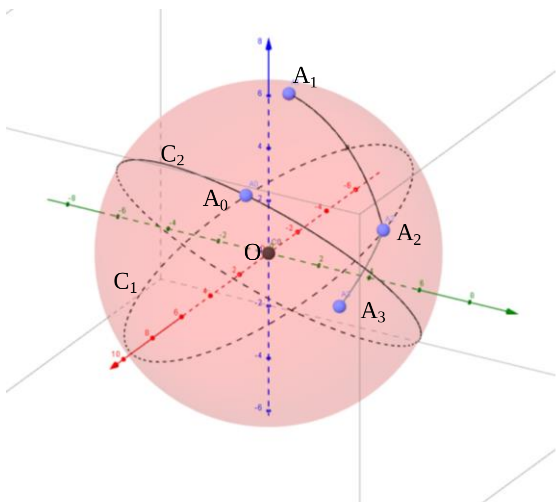


**Fig. 17. Spherical three position synthesis – procedure.**

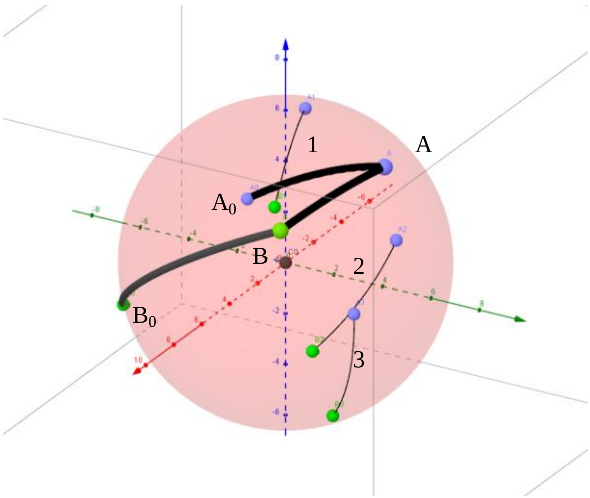


**Fig. 18. Spherical mechanism synthesis (GeoGebra©).**

The circles $C_1$ and $C_2$ correspond to the perpendicular bisectors. The intersection of both circles is the sought point $A_0$, equidistant to $A_1$, $A_2$ and $A_3$. The synthesis procedure is the same to find the position of point $B_0$, equidistant to $B_1$, $B_2$ and $B_3$. The resulting mechanism is shown with a bold line in Fig. 18.

In Fig. 19 the ROBERTS-CHEBYSHEV-theorem for a given spherical four-bar mechanism (*blue*) - moving on sphere S - is executed allowing to find two further four-bar linkages (*red*, *green*) that can generate the same coupler curve. Again, the synthesis procedure can be derived from the methods described in section 2.1.2 for planar mechanisms. It is for the first mechanism ((*) - $A_0A^*B^*B_0^*$):

- Point $A^*$
  = so that $A_0A^*KA$ is a **spherical parallelogram**
  = circle $D_1$: **intersection**(**sphere**(center K, radius $A_0A$),S)
  = circle $D_2$: **intersection**(**sphere**(center $A_0$, radius AK),S)
  = Construct point A*: **intersection**($D_1$,$D_2$)

- Point B*
  = so that projection of triangle (A*B*K) is **homothetic** to projection of triangle (AKB) in this order
  = circle $D_3$: **intersection**(**sphere**(center A*, radius AK·A*K / AB), S)
  = circle $D_4$: **intersection**(**sphere**(center K, radius BK ·A*K / AB), S)
  = Construct point B*: **intersection**($D_3$,$D_4$)

- Point $B_0$*
  = so that projection of triangle ($A_0B_0$*$B_0$) is **homothetic** to projection of triangle (AKB) in this order
  = circle $D_5$: **intersection**(**sphere**(center $A_0$, radius AK·$A_0B_0$ / AB), S)
  = circle $D_6$: **intersection**(**sphere**(center $B_0$, radius BK·$A_0B_0$ / AB), S)
  = Construct point $B_0$*: **intersection**($D_5$,$D_6$)

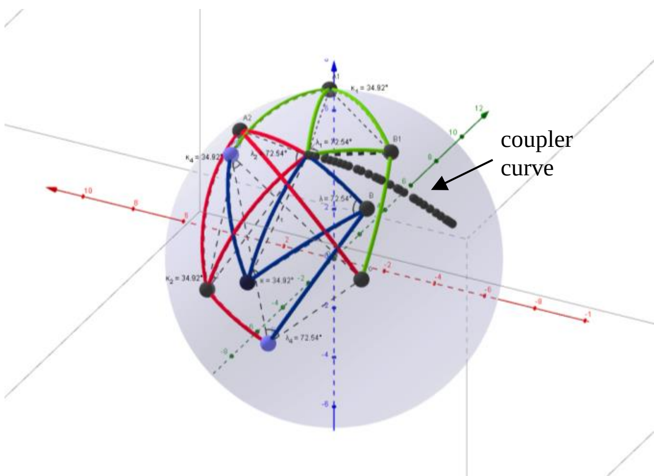


**Fig. 19. Spherical ROBERTS-CHEBYSHEV-theorem.**

A similar procedure can be derived for the second mechanism ((**) - $A_0$**A**B**$B_0$).

In summary, necessary design functionalities for spherical mechanisms are:

- Create spheres
- Perform Boolean operations, such as intersections
- Draw circular arcs
- Draw circles
- Use link length ratios to generate similar triangles (for ROBERTS-CHEBYSHEV-theorem)

GeoGebra© offers the possibility to display both, a 3D and a 2D window, the latter proving particularly useful for displaying sliders of the different parameters. Furthermore, 2D and 3D geometrical construction elements can be used within the same file.

### 2.4 Comparison

To compare the presented IGS and CAD tools with respect to the required time for mechanism synthesis and offered functionalities, the number of necessary operations for different tasks is listed in Tab. 1. Those tasks include: parametrization, angles and lengths transfer, perpendicular bisector sketching and finding the rotating point based on three points lying on a circular path. The number of required operations to draw parallel lines, to find intersection points or to draw circles/straight lines through given points does not vary in different programs. Therefore, these functionalities are not considered for comparison.

Mechanism synthesis in Cinderella© takes much longer compared to mechanism synthesis in GeoGebra©, because angle transfer, parametrization and bisector sketching extend the procedure time. It is worth mentioning that the 3D interface in GeoGebra© is a good and functional tool for spherical mechanism synthesis.

| | Parametrize | Transfer angles | Transfer lengths | Draw perpendicular bisectors | Find rotating point (position synthesis) |
|---|---|---|---|---|---|
| Cinderella© | 4 | 8 | 1 | 7 | 2 |
| GeoGebra© | 1 | 1 (2) | 1 | 3 | 2 |
| CATIA© | $1^{(*)}$ | 3 | 3 | - | 1 |
| Inventor© | 1 | 1 | 1 | - | 1 |

$^{(*)}$ here step "publishing" is not included

**Tab. 1. Required number of operations for different tasks.**

However, still a few useful functionalities are missing. For example it would be a strong advantage if the trace of a point generated by motion in GeoGebra© could be stored in a permanent parametrized way, instead of being regenerated for each change of the mechanism.

The approach for mechanism synthesis using the sketch mode within CAD systems is similar for CATIA© and Autodesk Inventor©. But angle and length transfer is more complicated in CATIA© because angles or lengths have to be measured first. Furthermore, publishing parameters and defining output features extend the procedure compared to Autodesk Inventor© where all parameters and construction elements can be directly globally used.

In general, CAD software provides extended capacities with respect to IGS (constraint solver, detailed 3D representation of parts), but requires more time for mechanism synthesis because of the complex interface. Besides, CAD software is less tolerant concerning mistakes within the design process. The engineer has to consider the synthesis steps precisely and has to choose the right references. Otherwise, the mechanism will not be adaptable later. Furthermore, dedicated tools for synthesis are not provided (e.g. displaying coupler curves).

In general, synthesis within a CAD system should be done after being familiar with mechanism theory and the implementation in an IGS, which is more intuitive than CAD. This is due to the fact that, in general, CAD systems were developed for detailed design more than for dimensional synthesis procedures. They require many details that are not useful for graphical synthesis methods and distract the designer from the task. For instance, skeletons are complex to build and the 2D synthesis procedures must be extended to 3D, although it is pointless most of the time. But once a deep knowledge of mechanism design is obtained, mechanism synthesis within CAD systems enables the engineer to interactively adapt mechanisms in order to optimize them. The results of changing parameters can be directly seen and the understanding of the motion behavior of the mechanism is enhanced.

## 3 Conclusion

In this paper the possibilities of two Interactive Geometry Software sketchers (IGS) (Cinderella©, GeoGebra©) and two CAD systems (Inventor Professional©, CATIA V5©) were compared. Relevant synthesis procedures were presented and explained in detail: three position synthesis, dead-center position synthesis and ROBERTS-CHEBYSHEV-theorem. All of them were reformulated as algorithms comprising a sequence of geometric operators. The algorithms were tested on the mentioned software types. Moreover, different synthesis examples were shown: a planar windscreen wiper and a spherical four-bar linkage. Based on these examples, necessary functionalities that software tools should provide were derived.

Although both types of software are capable to process the synthesis algorithms, the IGS appeared particularly efficient for fast, preliminary synthesis of linkages. Furthermore, the spherical mechanism could be easily treated with an IGS (GeoGebra©), thus showing the major interest of this category of software for mechanism dimensional synthesis.

Future CAD software should develop easier sketchers for mechanism synthesis, encouraging the use of mechanism skeletons and avoiding distracting the designer with useless details at this preliminary stage of the design process. For the moment, synthesis methods are lying in the designer's mind more than in the software, and many enhancements should be expected in the years to come.

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