# Preliminary Design and Analysis the Mobile Robot OpenWHEEL i3R

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**Résumé –** Dans cet article nous présentons un nouveau concept de robot mobile évoluant dans un environnement naturel : Open WHEEL i3R. Une étude préliminaire de conception permet de le caractériser. A travers cette étude, nous mettons en évidence la bonne adéquation entre l'architecture du robot et les mobilités requises par la tâche : rouler sur une surface quelconque et franchir des obstacles. Ensuite, nous élaborons les modèles cinématique, statique et dynamique utiles à la conception des composants mécaniques, à la planification de trajectoires, à l'analyse de stabilité et à la commande du robot.

**Mots clés :** robots mobiles, analyse structurale, modélisation cinématique, statique, dynamique.

**Abstract –** In this article we present a new concept of mobile robot that evolves in natural environment: Open WHEEL i3R. A preliminary design study allows characterizing it. Throughout this study, we underline the good adequacy between the robot architecture and the task required mobilities: rolling on an arbitrary surface and cross obstacles. Afterwards, we perform kinematic, static and dynamic models useful for the design of mechanical components, path planning, stability analysis and robot control.

**Key words:** mobile robots, structural analysis, kinematic, static, dynamic modelling.

## 1 Introduction

Mobile robots using wheels and evolving or moving in natural, uneven or discontinuous environments require a high degree of mobility in order to adapt to the geometrical variations of the ground and to cross obstacles. Evaluation of the design and modelling of efficient architectures and control strategies have a strong impact on their future behaviour and are subject to important research, e.g. [1, 2].

The authors of the present paper have already proposed in a previous works some innovative principles for climbing obstacles within the framework of the OpenWHEEL architecture for designing wheeled robots, keeping the efficiency of the wheels while improving mobility and static stability by a good compromise between climbing performance, complexity, stiffness and technological pragmatism. The OpenWHEEL 4sRR robot was presented in [3], with four motorized swing arm and the associated stable climbing process. The present paper deals with another agile robot named OpenWHEEL i3R (Figure 1), with four motorized wheels and only one central warping actuator. A 3D model has been created to validate the proposed climbing process [4]. First experimental results on a reduced model were also published [5].

The objective of this article is to give tools to the performance evaluation of the robot in order to optimize its design. An initial mobility analysis makes possible to check the adequacy of the retained architecture, challenging tasks such as moving in unstructured environments, in particular with the obstacle crossing. Kinematic, dynamic and static models of the robot are developed thereafter to simulate its behaviour and evaluate its performances. These models are of first importance in the development of control strategies. The considered performances are relative to static and dynamic stability, as to capacities of acceleration and crossing, with the view of the definition of stability margins.

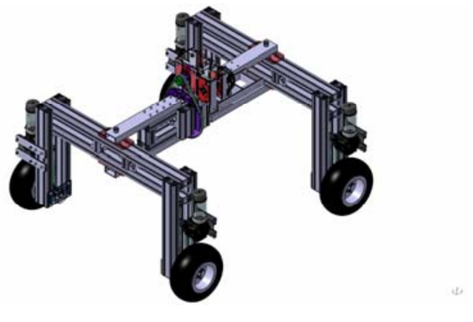

**Figure 1:** *CAD model of OpenWHEEL i3R*

## 2 Wheeled robotic systems

Mobile robots can be considered as a subpart of the vehicle family, more specially designed to challenge with complex reproducible tasks, often at low speed. For this aim, they usually have reactive behavior with the help of sensors for intern and extern perception, actuators, control laws and strategies for interpretation of sensory data and decision. The mechanical architecture can allow mono or multi-modes of locomotion. Internal mobilities can be passive (without actuators). Only the wheels are then motorized. On the contrary, some mobilities can be actuated (active robots). The addition of sensors also makes possible to adapt to the unknown factors and to changes of the ground (reactive robots).

The wheeled terrestrial propulsion is known to be a very energy-efficient way of moving, because energy is mainly used for propulsion and not lift [6]. Wheels are particularly fast on flat grounds but have difficulties to deal with obstacles and terrain discontinuities. Several robots offer a hybrid architecture by mounting wheels on/with legs [7, 8, 9, 10], combining more than two locomotion types [11], or presenting original articulated frames [12, 13] in order to locate and orientate wheels for specific purposes. However, this improvement is often obtained at the price of higher complexity, great number of joints, low stiffness and great number of wheels.

### *2.1 Mono-Mode Robots*

Mono-modes robots have only one mode of locomotion: rolling. For obstacle crossing, they classically rely on high wheels (wheel radius higher than the obstacle height), long travel suspension for keeping all wheels in contact with the ground and all-road tires. Many commercial robots are based on this architecture. Some have four wheels and are very close to car architecture [14].

### *2.2 Passive Multi-Mode Robots*

Others adopt only three wheels for permanent stability [15]. The majority of all-terrain mobile robots allow several modes of locomotion by the mean of internal motilities, creating suspensions with great displacement. The six-wheel architecture is quite an efficient solution for all-terrain vehicles [16] or planetary exploration robots such as Adam [17], Sojourner [18], Nexus 6 [19].

However, some are passive mobile robots, able to adapt to rough terrains or small obstacles, allowed by the architectural frame with no additional motorized degree of freedom or complex control laws. A usual suspension is the rocker-bogie suspension type (like Sojourner). Performance evaluation of several frames developed for planetary rovers is presented in [20]. Shrimp [21] is able to cross obstacles with height twice of the wheel diameter and adapt to concave or convex shapes. Micro5 [22] shows characteristics of simple and lightweight five drive-wheel vehicle with four wheels in the corners and one central supporting wheel improving climbing capabilities. The rover of the University of Rome [15] presents a mobile robotic structure with three orientation and driving wheels, mounted on symmetric leg-shaped supports, to allow the rover to cross small obstacles with a great stability.

### 2.3 Active Multi-Mode Robots

Most of multi-mode solutions incorporate active concepts by the mean of actuated internal mobilities. Most of them need motorized wheels instead of a central motor to simplify power transmission (distributed motorization). These modes are characterized by a succession of operations to carry out by the robot, which connects them up to a certain point to the concept of legged robots, such as Hylos [7].

Modes can be repositioning of the mass center (equilibrating) [23], or other locomotion modes such as combinations of rolling / climbing / peristalsis capabilities. The Marsokhod models allow controlling the inter-axles distance, with locomotion and a variation of the wheelbases which approach the crawling movement of caterpillar (peristaltic mode) [13]. Some use binary actuation to change the configuration of the suspension [24]. Active multi-mode vehicles find another utility in the military field for external robot-like applications similar to space exploration robots (locomotion with high mobility), e.g. robuROC [25], 6 wheel reconfigurable robot, with subroutines to dispatch the weight load on the six wheels for fast displacements in harsh environment.

A large number of mobile robots are specially designed for the task of step-climbing. Ref. [26] presents a holonomic omni-directional vehicle with changes of the body shape. The suspension incorporates passive joints but some sensors allow taking into account the modifications of the kinematic model for the control. A control method based on the variable kinematics model distributes the load among all wheels. The eight-wheeled robot OctalWheel [27] can challenge tasks such as climbing over obstacles and even stairs. OctalWheel is based on the same principle as Ibot 3000 [28], an armchair with four driving wheels for handicapped people which can go up and down staircases thanks to a rotating chassis and dynamic balancing capabilities. Helios [29] is also a 6 wheeled off-road vehicle with four low-pressure tires and two high-pressure tires with variable position, designed for powering a wheelchair or carrying tasks.

### 2.4 OpenWHEEL Paradigm

A previous paper [3] proposed a climbing mode and a modular architecture making possible to obtain high climbing capacities, while remaining slightly actuated and sufficiently generic to be easily adaptable and transposable on existing wheeled vehicles systems (e.g. quads). Its principal characteristics are gathered in Table 1 and shown in Figure 1. The global motorization is chosen to be distributed on the wheels, with one electric motor attached to each wheel, for compactness and genericity.

The OpenWHEEL platform is generic in the way that it should be understood as a modular assembly of various canonical components such as wheels (with attached electric motor), suspension mechanisms, axles, inter-axles mechanisms and other parts such as control microchips, sensors or communication devices.

| ***Choice for →*** | ***Ground contact type*** | ***Motorization*** | ***Mobility*** | ***Behavior*** |
|---|---|---|---|---|
| ***Some possible solutions →*** | $\boxed{\text{wheels}}$ | central | mono-mode | passive |
| | tracks | $\boxed{\text{distributed}}$ | $\boxed{\text{multi-mode}}$ | $\boxed{\text{slightly active}}$ |
| | legs | | | active |
| | adherence | | | reactive |

**Table 1:** *Conceptual Characteristics for OpenWHEEL*

The number of wheels in the OpenWHEEL architecture should be understood as free *a priori.* Among terrestrial vehicles, six-wheel architectures are not very common because they are expensive and bring steering problems. The vast majority of commercial vehicles have only four wheels. It should be noted that only three wheels are required to ensure permanent stability but the three-wheel architecture is not widespread, probably because of non-symmetry and dynamic instability. However, the idea of static stability on three wheels is developed for a four-wheel vehicle. The wheel not used for stability is called "the exploring wheel". This wheel is dedicated to explore above the obstacle and then should be put on it to offer a new support point. After that, another wheel becomes the exploring wheel and the process can go on.

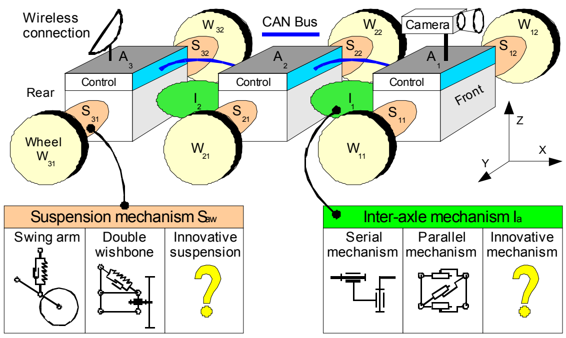


**Figure 2:** *OpenWHEEL architecture*

## 3 Mobility analysis

In this section, mobility analysis aims to verify the adequacy between required mobility for the task, robot mobility and the number of actuated joints. The presented robot (Figure 3) has to move on a ground surface in the three-dimensional space along a trajectory that can be defined as a curve on this surface. These task specifications imply that two degrees of mobility are required: longitudinal and lateral motions. In order to guarantee good controllability and stability, this task must be accomplished under the following constraints:

- the four wheels remain in contact with the ground, except for crossing manoeuvres,
- rolling without slipping wheel – ground contact condition must be compatible with robot kinematics.

The degree of mobility, or the configuration space dimension of the robot, is denoted by *M*. It indicates also the minimal number of motorized joints necessary to control robot motions. It can change in crossing manoeuvres when one of the four wheels leaves the ground.

The first step of this analysis consists in determining the degree of mobility of each joint, particularly for wheel – ground contact "joint". If rolling without slipping condition is assumed, this type of "joint" of torus shaped wheel has 3 degrees of mobility. Each revolute joint has 1 degree of mobility.

Let's consider now a module formed by a chassis and two wheels. Suspension mechanisms aren't taken into account. Each wheel has 1 degree of mobility with the chassis corresponding to a revolute joint. The mobility of a single module is obtained using the following formula [30]:

$$M_1 = \sum_{i=1}^{n} f_i - \chi_1 \qquad (1)$$

where $n = 4$ is the number of joints of a module and $\chi$ is the degree of connectivity of the associated open mechanism chain. $f_i$ is the degree mobility of the *i*th joint ($f_i = 1$ for revolute joints and $f_i = 3$ for wheel – ground contact joints). For arbitrary oriented revolute joint axes, $\chi = 6$. The degree of connectivity falls when the revolute joint axis become parallel: $\chi = 5$. In the first case the mobility of a module is given by:

$$M_1 = (2 \times 3 + 2 \times 1) - 6 = 2 \qquad (2)$$

In our case, revolute joint axes are parallel and then:

$$M_1 = (2 \times 3 + 2 \times 1) - 5 = 3 \qquad (3)$$

The module degree of mobility corresponds to the number of independent motions that has a chassis relatively to the ground. Therefore, each chassis can be seen as a part related to the ground by a 3-degree-of-mobility "complex" joint: 2 rotations and 1 translation.

Let now connect the two modules to the inter-axle mechanism. This introduces two additional links and three revolute joints. If the connection between the ground and the chassis is directly integrated by the complex joint defined previously, the mechanism is formed by 4 links with a closed chain. The mobility of robot is given by:

$$M = \sum_{i=1}^{k} f_i - \chi \qquad (4)$$

where $k = 5$: 2 complex and 3 revolute joints. The degree of connectivity, $\chi$, associated to the open chain mechanism is equal to 6.

$$M = (2 \times 3 + 3 \times 1) - 6 = 3 \qquad (5)$$

At the sight of this result, it can be stated that the mechanism has an extra degree of mobility to accomplish the task of rolling on the ground surface along a curve. Indeed, only two degrees of mobility are required for this task. The relative orientation between the two modules can be controlled. In addition, by using four motorized wheels the number of actuated joints is greater than needed.

In climbing phase, one of the four wheels leaves the ground. Let denote by $M_2$ the degree of mobility of the module having only one wheel in contact with the ground. This module is an open chain mechanism and its degree of mobility is simply given by:

$$M_2 = \sum_{i=1}^{l} f_i \qquad (6)$$

where $l = 2$ is the number of joints: 1 revolute joint and 1 wheel – ground contact joints. The wheel which is no longer in contact with the ground is ignored. Its rotation doesn't affect the robot motion.

$$M_2 = 1 + 3 = 4 \qquad (7)$$

$M_2$ is also to the number of independent motions that has the corresponding chassis relatively to the ground. Therefore, this chassis can be seen as a part related to the ground by a 4-degree-of-mobility "complex" joint. The degree of mobility of the robot in climbing phase can be determined as in the previous subsection:

$$M=\sum_{i=1}^{k} f_i-\chi=(3+4+3\times 1)-6=4 \qquad (8)$$

This result shows that the mechanism degree of mobility allows accomplishing the task of rolling on the ground surface along a curve while elevating a wheel for climbing. One extra degree of mobility remains. It can be used to control the relative orientation between the two modules. In this case, the robot is properly and minimally actuated since the central inter-axle joint is motorized.

The presented analysis doesn't take into account possible mechanism singularities that affect locally the degree of connectivity.

## 4 Geometric parameters

The vehicle is formed by eight bodies. To each body $i$, it is attached a frame $R_i(O_i,\vec{x}_i,\vec{y}_i,\vec{z}_i)$, $i=0\ldots 8$. $R_0$ is the fixed frame of reference. The global position and orientation of the vehicle are defined by the configuration of the frame $R_1$ relatively to $R_0$. The other geometric variables are intrinsic to the vehicle. The position variables are the coordinates of point $O_1$ in $R_0$.

$$\overrightarrow{O_0O_1}=[x_{O1} \quad y_{O1} \quad z_{O1}]^T_{R_0}$$

The orientation of the frame $R_1$ is given by three angular variables:

$\psi=\angle(\vec{x}_0,\vec{x}_{1a})=\angle(\vec{y}_0,\vec{y}_{1a})$ yaw angle

$\theta=\angle(\vec{z}_{1a},\vec{z}_{1b})=\angle(\vec{x}_{1a},\vec{x}_{1b})$ pitch angle

$\phi=\angle(\vec{y}_{1b},\vec{y}_1)=\angle(\vec{z}_{1b},\vec{z}_1)$ roll angle

$R_{1a}(O_1,\vec{x}_{1a},\vec{y}_{1a},\vec{z}_0)$ and $R_{1b}(O_1,\vec{x}_{1b},\vec{y}_{1a},\vec{z}_{1b})$ are two intermediate frames. Intrinsic variables are the revolute joint angles:

$\theta_{12}=\angle(\vec{y}_1,\vec{y}_2)=\angle(\vec{z}_1,\vec{z}_2)$

$\theta_{13}=\angle(\vec{x}_1,\vec{x}_3)=\angle(\vec{y}_1,\vec{y}_3)$

$\theta_{24}=\angle(\vec{x}_2,\vec{x}_4)=\angle(\vec{y}_2,\vec{y}_4)$

To these variables, angular positions of each wheel are added:

$\theta_{35}=\angle(\vec{x}_3,\vec{x}_5)=\angle(\vec{z}_3,\vec{z}_5);\theta_{36}=\angle(\vec{x}_3,\vec{x}_6)=\angle(\vec{z}_3,\vec{z}_6);$

$\theta_{47}=\angle(\vec{x}_4,\vec{x}_7)=\angle(\vec{z}_4,\vec{z}_7);\theta_{48}=\angle(\vec{x}_4,\vec{x}_8)=\angle(\vec{z}_4,\vec{z}_8).$

Geometric constants, or design variables, are defined in terms of distances between joint centres.

$\overrightarrow{O_1O_3}=-l_3\vec{x}_1-h_3\vec{z}_1\,;\overrightarrow{O_2O_4}=l_4\vec{x}_2-h_4\vec{z}_2$

$\overrightarrow{O_3O_5}=-e_5\vec{y}_3-h_5\vec{z}_3\,;\overrightarrow{O_3O_6}=e_5\vec{y}_3-h_5\vec{z}_3$

$\overrightarrow{O_4O_7}=-e_7\vec{y}_4-h_7\vec{z}_4\,;\overrightarrow{O_5O_8}=e_7\vec{y}_4-h_7\vec{z}_4$

In order to simplify robot design, the centres of revolute joints $(O_1,O_2,O_3)$ are aligned. The following relations between geometric parameters are adopted.

$l_3=l_4\,;h_3=h_4=0\,;\,e_5=e_7\,;\,h_7=h_5$

The configuration of each reference can be defined by a $4\times 4$ homogeneous matrix.

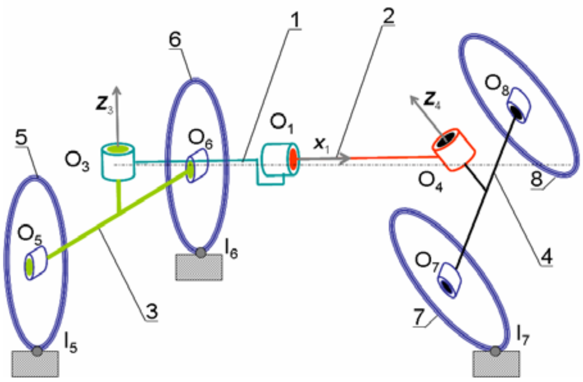


**Figure 3:** *Kinematic diagram of OpenWHEEL i3R*

## 5 2D kinematic and dynamic analyses

In this section kinematics analysis is performed when the robot involves on a plane surface. The four wheels are in contact with the ground. This analysis aims to determine the vehicle velocity and acceleration fields while wheel motions are controlled. To this end, non slipping assumption is adopted. Mobility analysis showed that the vehicle has only three degrees of mobility. Therefore, by using four motorized wheels the vehicle has a redundant actuator. Wheel motions can not be independent. Dynamic equations of motion, relating applied torques to vehicle velocity and acceleration are derived using Newton-Euler formulation.

### *5.1 Kinematic analysis*

In two dimensional robot kinematics, pitch and roll angles are not considered and remain at zero. Each wheel has a torus form with $R$ and $r$ as major and minor radii. Wheel center velocities are expressed using the non slipping assumption:

$$\vec{V}_{O_5\in R_5/R_0}=(R+r)\dot{\theta}_{35}\vec{x}_3 \qquad (9a)$$

$$\vec{V}_{O_6\in R_6/R_0}=(R+r)\dot{\theta}_{36}\vec{x}_3 \qquad (9b)$$

$$\vec{V}_{O_7\in R_7/R_0}=(R+r)\dot{\theta}_{47}\vec{x}_4 \qquad (9c)$$

$$\vec{V}_{O_8\in R_8/R_0}=(R+r)\dot{\theta}_{48}\vec{x}_4 \qquad (9d)$$

Body 3 has rigid body motion. In addition, points $O_5$ and $O_6$ belong also to body 3. Therefore, the velocity of the point $O_3$ can be expressed in two different ways:

$$\vec{V}_{O_3\in R_3/R_0}=\vec{V}_{O_5\in R_5/R_0}+\vec{\Omega}_{R_3/R_0}\times e_5\vec{y}_3$$

$$\vec{V}_{O_3\in R_3/R_0}=\vec{V}_{O_6\in R_6/R_0}-\vec{\Omega}_{R_3/R_0}\times e_5\vec{y}_3$$

The velocity of the point $O_3$ can be easily deduced as well as the relation between wheels and vehicle angular variables:

$$\vec{V}_{O_3\in R_3/R_0}=\frac{R+r}{2}(\dot{\theta}_{35}+\dot{\theta}_{36})\vec{x}_3 \qquad (10a)$$

$$(\dot{\psi}+\dot{\theta}_{13})=\frac{R+r}{2e_5}(\dot{\theta}_{35}-\dot{\theta}_{36}) \qquad (10b)$$

Time differentiation of equations (10a) and (10b) gives acceleration relations:

$$\vec{\Gamma}_{O_3\in R_3/R_0} = \frac{R+r}{2}\left[(\ddot{\theta}_{35}+\ddot{\theta}_{36})\vec{x}_3 + (\dot{\theta}_{35}+\dot{\theta}_{36})(\dot{\theta}_{13}+\dot{\psi})\vec{y}_3\right] \quad (10c)$$

$$(\ddot{\psi}+\ddot{\theta}_{13}) = \frac{R+r}{2e_5}(\ddot{\theta}_{35}-\ddot{\theta}_{36}) \quad (10d)$$

By the same way, one can obtain velocity and acceleration relations of the body 4.

$$\vec{V}_{O_4\in R_4/R_0} = \frac{R+r}{2}(\dot{\theta}_{47}+\dot{\theta}_{48})\vec{x}_4 \quad (11a)$$

$$(\dot{\psi}+\dot{\theta}_{24}) = \frac{R+r}{2e_5}(\dot{\theta}_{47}-\dot{\theta}_{48}) \quad (11b)$$

$$\vec{\Gamma}_{O_4\in R_4/R_0} = \frac{R+r}{2}\left[(\ddot{\theta}_{47}+\ddot{\theta}_{48})\vec{x}_4 + (\dot{\theta}_{47}+\dot{\theta}_{48})(\dot{\theta}_{24}+\dot{\psi})\vec{y}_4\right] \quad (11c)$$

$$(\ddot{\psi}+\ddot{\theta}_{24}) = \frac{R+r}{2e_5}(\ddot{\theta}_{47}-\ddot{\theta}_{48}) \quad (11d)$$

Points $O_3$ and $O_4$ belong respectively to the bodies 1 and 2. In 2D kinematics, these two bodies can be considered as one rigid body. This allows writing differently the velocity expression of point $O_4$ :

$$\vec{V}_{O_4\in R_4/R_0} = \vec{V}_{O_3\in R_3/R_0} + \dot{\psi}\,\vec{z}_0 \times (l_3+l_4)\vec{x}_1 \quad (12)$$

Equations (10a), (11a) and (12) give us a vector relation between the yaw and wheel angular velocities:

$$\frac{R+r}{2}(\dot{\theta}_{47}+\dot{\theta}_{48})\vec{x}_4 = \frac{R+r}{2}(\dot{\theta}_{35}+\dot{\theta}_{36})\vec{x}_3 + (l_3+l_4)\dot{\psi}\,\vec{y}_1 \quad (13)$$

The projection of (13) on $\vec{y}_1$ gives the yaw velocity:

$$\dot{\psi} = \frac{R+r}{2(l_3+l_4)}\left[\sin(\theta_{24})(\dot{\theta}_{47}+\dot{\theta}_{48}) - \sin(\theta_{13})(\dot{\theta}_{35}+\dot{\theta}_{36})\right] \quad (14)$$

The projection of (13) on $\vec{x}_1$ gives an intrinsic nonholonomic mechanism constraint:

$$\cos(\theta_{24})(\dot{\theta}_{47}+\dot{\theta}_{48}) - \cos(\theta_{13})(\dot{\theta}_{35}+\dot{\theta}_{36}) = 0 \quad (15a)$$

Time differentiation of equation (15a) gives an acceleration constraint:

$$\begin{aligned}\cos(\theta_{24})(\ddot{\theta}_{47}+\ddot{\theta}_{48}) - \cos(\theta_{13})(\ddot{\theta}_{35}+\ddot{\theta}_{36}) &\; - \\ \dot{\theta}_{24}\sin(\theta_{24})(\dot{\theta}_{47}+\dot{\theta}_{48}) &\; + \\ \dot{\theta}_{13}\sin(\theta_{13})(\dot{\theta}_{35}+\dot{\theta}_{36}) &\; = 0\end{aligned} \quad (15b)$$

The angular and linear velocities of the frame $R_1$ are obtained from previous relations:

$$\vec{\Omega}_{R_1/R_0} = \frac{R+r}{2(l_3+l_4)}\left[\sin(\theta_{24})(\dot{\theta}_{47}+\dot{\theta}_{48}) - \sin(\theta_{13})(\dot{\theta}_{35}+\dot{\theta}_{36})\right]\vec{z}_0 \quad (16a)$$

$$\begin{aligned}\vec{V}_{O_1\in R_1/R_0} = &\; \frac{R+r}{2}(\dot{\theta}_{35}+\dot{\theta}_{36})\vec{x}_3 \\ + &\; \frac{l_3(R+r)}{2(l_3+l_4)}\left[\sin(\theta_{24})(\dot{\theta}_{47}+\dot{\theta}_{48}) - \sin(\theta_{13})(\dot{\theta}_{35}+\dot{\theta}_{36})\right]\vec{y}_1\end{aligned} \quad (16b)$$

### *5.2 Dynamic analysis*

Dynamic analysis consists in deriving the vehicle equations of motion under four input wheel torques. This allows evaluating acceleration capacities for given masses and body inertias. To this end, Newton-Euler formulation is adopted by using appropriate projections of vector relations. Referred to mobility analysis, vehicle dynamics is given by three equations of motion. External applied forces are the contact reactions at points $I_5$, $I_6$, $I_7$ and $I_8$. These forces are denoted by :

$$\vec{F}_{05} = [X_{05} \quad Y_{05} \quad Z_{05}]^T_{R_3};\; \vec{F}_{06} = [X_{06} \quad Y_{06} \quad Z_{06}]^T_{R_3}$$

$$\vec{F}_{07} = [X_{07} \quad Y_{07} \quad Z_{07}]^T_{R_4};\; \vec{F}_{08} = [X_{08} \quad Y_{08} \quad Z_{08}]^T_{R_4}$$

$X_{0i}, Y_{0i}, Z_{0i}\,; i = 5\ldots8$ are respectively longitudinal, lateral and normal contact reactions. In 2D analysis, normal reactions are not considered. The wheel input torques are denoted by:

$$\vec{C}_{35} = C_5\vec{y}_3;\; \vec{C}_{36} = C_6\vec{y}_3;\vec{C}_{47} = C_7\vec{y}_4;\vec{C}_{48} = C_8\vec{y}_4;$$

The four wheels are identical and have the same mass and inertia momentum $J_r$ about their rotation axis. Angular accelerations the wheels are governed by the following equations:

$$\ddot{\theta}_{35} = (C_5 - (R+r)X_{05})/J_r \quad (17a)$$

$$\ddot{\theta}_{36} = (C_6 - (R+r)X_{06})/J_r \quad (17b)$$

$$\ddot{\theta}_{47} = (C_7 - (R+r)X_{07})/J_r \quad (17c)$$

$$\ddot{\theta}_{48} = (C_8 - (R+r)X_{08})/J_r \quad (17d)$$

Bodies 3 and 4 have the same mass inertia parameters. Their inertia momentum in $O_3$ and $O_4$ about $\vec{z}_3$ and $\vec{z}_4$ is denoted by $J_m$. The following dynamic equations can be written:

$$\ddot{\theta}_{13} + \ddot{\psi} = e_5(X_{05} - X_{06})/J_m \quad (18a)$$

$$\ddot{\theta}_{24} + \ddot{\psi} = e_5(X_{07} - X_{08})/J_m \quad (18b)$$

By using equations (10d) and (11d), longitudinal force can be related to applied torques as follows:

$$(X_{05} - X_{06}) = \frac{(R+r)J_m}{e_5^2 J_r + (R+r)^2 J_m}(C_5 - C_6) \quad (19a)$$

$$(X_{07} - X_{08}) = \frac{(R+r)J_m}{e_5^2 J_r + (R+r)^2 J_m}(C_7 - C_8) \quad (19b)$$

Dynamic equations can be expressed according to applied torques:

$$\ddot{\theta}_{13} + \ddot{\psi} = \frac{e_5(R+r)}{e_5^2 J_r + (R+r)^2 J_m}(C_5 - C_6) \quad (20a)$$

$$\ddot{\theta}_{24} + \ddot{\psi} = \frac{e_5(R+r)}{e_5^2 J_r + (R+r)^2 J_m}(C_7 - C_8) \quad (20b)$$

Now, the dynamic equilibrium of the body formed by the union of the two bodies 1 and 2 is considered. Its inertia is neglected. Therefore, dynamic equations become as in the static case. Forces applied by bodies 3 and 4 on, respectively, bodies 1 and 2 are denoted by:

$$\vec{F}_{31} = [X_{31}\, Y_{31}\, Z_{31}]^T_{R_1} \quad \text{and} \quad \vec{F}_{42} = [X_{42}\, Y_{42}\, Z_{42}]^T_{R_1}$$

Force and momentum equilibrium equations in the plane and about $\vec{z}_0$ are:

$$\left.\begin{aligned}X_{31} + X_{42} &= 0 \\ Y_{31} + Y_{42} &= 0 \\ (l_3 + l_4)Y_{42} &= 0\end{aligned}\right\} \Rightarrow \begin{aligned}X_{31} &= -X_{42} \\ Y_{31} &= Y_{42} = 0\end{aligned}$$

Bodies 3 and 4 centers of mass are assumed to be, respectively, at $O_3$ and $O_4$. The accelerations of these two points can be related the forces applied to rear and front axles formed, respectively, by bodies 3, 5 and 6, and bodies 4, 7 and 8. Two dynamic equations can be written by projecting accelerations on $\vec{x}_3$ and $\vec{x}_4$ :

$$\vec{\Gamma}_{O_3 \in R_3/R_0} \cdot \vec{x}_3 = \frac{(R+r)}{2}(\ddot{\theta}_{35} + \ddot{\theta}_{36}) \qquad (21a)$$

Therefore:

$$\frac{R+r}{2}(\ddot{\theta}_{36} + \ddot{\theta}_{35}) = \frac{X_{05} + X_{06} - X_{31}\cos(\theta_{13})}{m_m + 2m_r} \qquad (21b)$$

where, $m_r$ is the mass of a wheel and $m_m$ is the mass of bodies 3 and 4. By the same way, it can be obtained:

$$\vec{\Gamma}_{O_4 \in R_4/R_0} \cdot \vec{x}_4 = \frac{R+r}{2}(\ddot{\theta}_{47} + \ddot{\theta}_{48}) \qquad (22a)$$

$$\frac{R+r}{2}(\ddot{\theta}_{47} + \ddot{\theta}_{48}) = \frac{X_{07} + X_{08} - X_{31}\cos(\theta_{24})}{m_m + m_r} \qquad (22b)$$

Equations (15b), (17a-d), (19a), (19b), (21b), (22b) lead to a linear system of five equations from which unknown forces: $X_{05}$, $X_{06}$, $X_{07}$, $X_{08}$ and $X_{31}$ are obtained. Thereafter, dynamic equations are obtained by replacing reaction forces in (17a-d). Vehicle motion is completely determined for given wheel angular acceleration and by using kinematic relations. The dynamic model developed above is useful for mechanical and control design of the robot.

## 6 Static analysis

Static analysis aims to determine the inter axle joint torque needed to lift up a wheel as well as wheel torques that maintain vehicle equilibrium. Vehicle static stability can be characterized by normal wheel ground contact forces. In order to simplify this analysis, it is performed for $\theta_{13} = \theta_{24} = 0$.

The vehicle is submitted to gravity. The torque in the inter axle allows the landing off of one wheel e.g. 8. The vehicle equilibrium results from ground contact forces, wheel torques and inter axle torque. There are 13 scalar static unknown forces and torques. These unknowns are given by:

$$\vec{F}_{0i} = X_{0i}\vec{x}_0 + Y_{0i}\vec{y}_0 + Z_{0i}\vec{z}_0 \qquad i = 5,6,7$$

$$\vec{C}_{35} = C_{35}\vec{y}_3, \vec{C}_{36} = C_{36}\vec{y}_3, \vec{C}_{47} = C_{47}\vec{y}_4$$

$$\vec{C}_{12} = C_{12}\vec{x}_1$$

In order to determine these unknowns, 13 scalar equations of equilibrium can be written as follows:

By isolating the following systems and using appropriate projections of their static equilibrium condition, we obtain:

System $S_1$ formed by bodies (1), (2), (3), (4), (5), (6), (7) and (8), one can write the equilibrium equations in vector form:

$$\sum \vec{F}_{ext/S_1} = \vec{0}$$

$$\sum \vec{M}_{ext/S_1}(I_5) = \vec{0}$$

These equations lead to 6 scalar equalities:

$$X_{05} + X_{06} + X_{07} = 0 \qquad (23a)$$

$$Y_{05} + Y_{06} + Y_{07} = 0 \qquad (23b)$$

$$Z_{05} + Z_{06} + Z_{07} - 2mg = 0 \qquad (23c)$$

$$(\overrightarrow{I_5I_6} \times \vec{F}_{06} + \overrightarrow{I_5I_7} \times \vec{F}_{07} + \overrightarrow{I_5O_1} \times -2mg\vec{z}_0) \cdot \vec{x}_0 = 0 \qquad (23d)$$

$$(\overrightarrow{I_5I_6} \times \vec{F}_{06} + \overrightarrow{I_5I_7} \times \vec{F}_{07} + \overrightarrow{I_5O_1} \times -2mg\vec{z}_0) \cdot \vec{y}_0 = 0 \qquad (23e)$$

$$(\overrightarrow{I_5I_6} \times \vec{F}_{06} + \overrightarrow{I_5I_7} \times \vec{F}_{07} + \overrightarrow{I_5O_1} \times -2mg\vec{z}_0) \cdot \vec{z}_0 = 0 \qquad (23f)$$

By isolating successively each wheel in contact with the ground, bodies (5), (6) and (7), three other equations are obtained from the momentum equilibrium around wheel axes:

$$C_{35} - RX_{05} = 0 \qquad (24a)$$

$$C_{36} - RX_{06} = 0 \qquad (24b)$$

$$C_{47} - [R + r\cos(\alpha)]X_{07} = 0 \qquad (24c)$$

System $S_2$ formed by bodies (3), (5) and (6):

$$\sum \vec{M}_{ext/S_2}(O_3) \cdot \vec{z}_3 = 0 \qquad (25)$$

System $S_3$ formed by bodies (1), (3), (5) and (6):

$$\sum \vec{M}_{ext/S_3}(O_1) \cdot \vec{x}_1 = 0 \qquad (26)$$

System $S_4$ formed by bodies (2), (4), (7) and (8):

$$\sum \vec{M}_{ext/S_4}(O_1) \cdot \vec{x}_1 = 0 \qquad (27)$$

System $S_5$ formed by bodies (4), (7) and (8):

$$\sum \vec{M}_{ext/S_5}(O_4) \cdot \vec{z}_4 = 0 \qquad (28)$$

Equations (23a-f), (24a-c) and (25-28) form a linear system according to static unknown forces and torques. In order to solve this system, the coordinates of wheel – ground contact points, $I_5$, $I_6$ and $I_7$ have to be determined as function of inter axle angle. At this preliminary design stage, the ground surface is considered as horizontal plane. In addition, yaw and roll angles ($\psi$ and $\phi$) remain at zero and $\overrightarrow{O_0O_1} = [0 \quad 0 \quad z_{O1}]^T_{R_0}$. The centre of mass altitude, $z_{O1}$, and the pitch angle, $\theta$, vary as represented in Figure 3 as function of inter axles angle $\theta_{12}$.

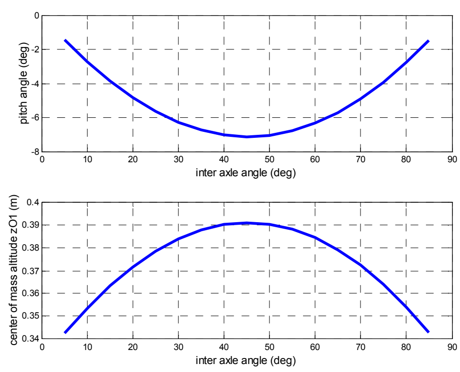


**Figure 4:** *centre of mass altitude* $z_{O1}$ *and pitch angle* $\theta$ *as functions of inter axle angle* $\theta_{12}$

Wheel – ground contact forces, longitudinal, lateral and normal, are represented in Figure 4. This type of graphics can be used to perform stability analysis, by examining normal forces, and detect slipping at contact points.

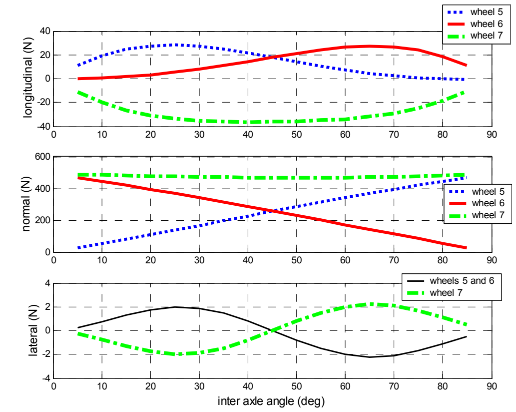


**Figure 5:** *wheel – ground contact forces*

The inter axle joint torque needed to maintain the mobile robot equilibrium when the wheel 8 leaves the ground is represented in Figure 5. It varies almost linearly according to inter axle angle $\theta_{12}$. It reaches its maximal magnitude about 150 N.m for $\theta_{12}$ near 0 or 90 deg. This analysis allows dimensioning inter axle actuator and transmission.

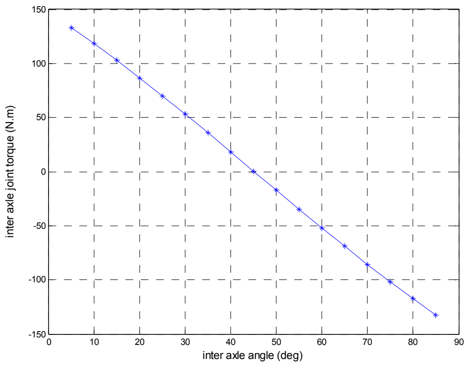


**Figure 6:** *inter axle joint torque*

## 7 Conclusion

In this article, a new concept of mobile robot evolving in unstructured environment is presented. The OpenWHEEL i3R uses a serial inter-axle mechanism. This concept was approved in term of adequacy between required mobilities, mechanism mobilities and actuated joints. 2D kinematic and dynamic modelling gives the basic relations to perform mechanical design, trajectory planning and robot control. Static analysis allows verifying stability conditions and determining inter axle torque needed to lift one wheel in climbing manoeuvres. Future work will focus on the implementation of the presented models on the robot control system and 3D dynamic modelling.

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