A semi-analytical stiffness model of parallel robots from the Isoglide family via the sub-structuring principle

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Abstract—[1] *this paper presents a stiffness study of two parallel robots from the Isoglide family, which have decoupled motions and are isotropic in translation. Stiffness is an important parameter for robot design and its computation requires often a long calculation time and a high computer capacity. In this paper a new method for stiffness calculation is presented. It is based on a semi-analytic stiffness model obtained by decomposing the robot into elementary legs and re-assembling their stiffness matrices. This method allows to obtain stiffness in a short computing time. Based on this method the stiffness maps of a real Isoglide prototype are developed. From the stiffness maps, the effects of the fourth DOF and the fourth leg were pointed out.*

Keywords: Isoglide parallel robot, stiffness maps, modular design, sub-structuring, semi-analytic model

I. Introduction

A parallel mechanism can be defined as mechanism with closed kinematic chain, made up of an end-effector with N degrees of freedom and a fixed base, connected to each other by at least two kinematic chains, the motorization being carried out by N actuators [1]. It presents many advantages with regard to serial mechanisms (higher stiffness/mass ratio, higher load,…). However it has a complex kinematics. This complexity is mainly due to the coupling of its motions, from where the need of the decoupling. Usually parallel robots have a high stiffness, which is an important parameter for the characterization of their performance. If the stiffness of the links and joints are inadequate, external loads can cause large deflexion in the mechanical parts. This paper presents a stiffness study based on the sub-structuring principle of 3 and 4 degrees of freedom (DOF) parallel robots from the Isoglide family.

II. Robots presentation

A parallel robot is characterised by its forward differential kinematic model which establishes a relation between the infinitesimal variation of joint coordinates $[dq]$ and the infinitesimal variation of the operational coordinates $[dX]$. This relation can be expressed in matrix form as:

$$[dX] = [J][dq] \tag{1}$$

$[J]$ is known as the Jacobian matrix. A parallel robot is[2]:

- fully isotropic if $[J]$ is proportional to the identity matrix.
- a parallel robot with uncoupled motions if $[J]$ is diagonal.
- a parallel robot with decoupled motions if $[J]$ is triangular.
- a parallel robot with coupled motions, in other cases.

The Isoglide [3] family gathers parallel robot from 2 to 6 DOF. All these robots are with decoupled motions and are isotropic in translations.Two robots from the Isoglide family will be treated in this paper. The first one is named Isoglide3-T3. It is made of three legs (denoted L_i) and has 3 DOF in translations (T3). The second one is called Isoglide4-T3R1 and is an enhanced version of Isoglide3-T3 with a supplementary leg for adding one rotation mobility (R1) [4]. These robots are modular parallel mechanisms and are two representatives of the extended Isoglide family of parallel robots [3]. Isoglide4-T3R1 is represented in Fig 1. Isoglide3-T3 can be seen in white while the fourth leg is in gray.

The forward differential kinematic model of the Isoglide4-T3R1 is:

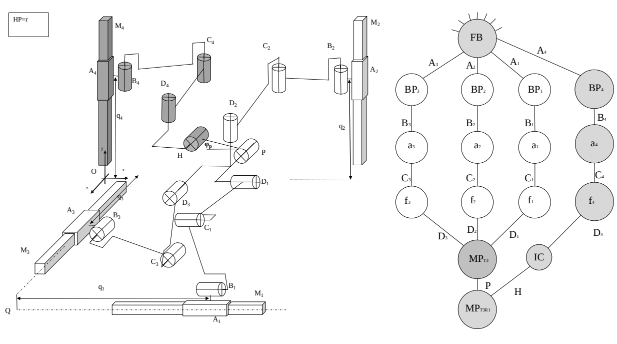

Fig.1: Kinematic scheme and graph of the Isoglide4-T3R1

$$\begin{bmatrix} dx_P \\ dy_P \\ dz_P \\ d\varphi_P \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -r\cos\varphi_P & 0 & r\cos\varphi_P \end{bmatrix} \begin{bmatrix} dq_1 \\ dq_2 \\ dq_3 \\ dq_4 \end{bmatrix}. \tag{2}$$

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where, r is the moving platform length defined as the distance between H and P and φ_P is its orientation angle.

The Jacobian matrix of the Isoglide4-T3R1 shown in (2) is triangular. Consequently the robot has decoupled motions. The sub-matrix which maps translations velocities to the first three actuator velocities of the Isoglide3-T3 is the identity matrix. The Isoglide3-T3 is fully isotropic and the Isoglide4-T3R1 is isotropic in translation. The integration of (2) gives the forward kinematic model from which it is possible to get all geometric parameters of the robot notably the leg folding angles and the arm orientations.

III. Problem setting

The problem consists in calculating the deformation of the Isoglide in all its workspace. Thus, it is a matter of applying the Hooke's law:

$$[K][X]=[F] \tag{3}$$

on the Isoglide that becomes a structure if its actuators are locked. Two approaches for the stiffness calculation can be applied, considering deformation located in actuators and assuming each one to a spring [11], considering links deformations where various methods of computation can be applied:

- assuming the components as linear and torsional springs (lumped stiffness) [6,7,12],
- study of the displacement considering each component as a beam, using Euler-Bernoulli beam model for instance (distributed stiffness) [8- 10, 17],
- numerical simulation via the Finite Element Method (FEM) [5, 13-16, 19]

The lumped stiffness method gives acceptable result in a quick computation time, but it is very hypothetic. The second method gives good result if the components are beams. The FEM has the advantage of giving the best result since the Isoglide is modelled with its true shape and dimension. The only error in this method is due to the discretisation of the continuous space [18]. On the other hand, this method has the disadvantage that it requires an extensive computation time [9].

In this paper, we present a new method for the stiffness calculation based on the sub-structuring principle. This method has the advantage that it gives the same results as FEM with shorter calculation time and few computer capacity and memory.

IV. Stiffness calculation

The methodology used in this paper benefits from the modular design of the robot. First we calculate the stiffness matrix of an isolated leg. The Isoglide3-T3 stiffness matrix is deduced by the assembly of three legs. Isoglide4-T3R1 structure is the assembly of two substructures, the Isoglide3-T3 structure and the fourth leg L_4 structure.

A. Isolated leg stiffness matrix

Because of revolute joints, the leg can only bear a force parallel to its revolute joint axes, or a moment which does not have a component parallel to these axes. The compliance matrix of the leg maps three loads to three displacements and is a 3x3 matrix.

The leg is made from two solids connected by a revolute joint (Fig 2). The solid BC is called arm and denoted a_i. The solid CD is called forearm and denoted f_i. Arm and forearm are two sub-structures serially connected. The compliance matrix of the leg is consequently the sum of the arm and the forearm compliance matrices reduced to point D (center of revolute joint D)

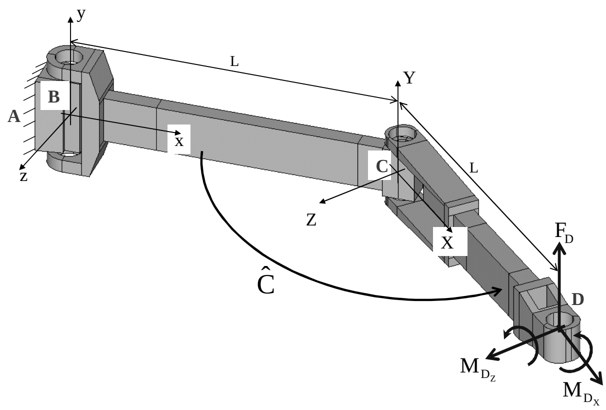

Fig.2: CAD model of an isolated leg

The compliance matrix of the leg is:

$$[S]=\left[{}^{f}S\right]+[U]^{T}\left[{}^{a}S\right][U], \tag{4}$$

where $\left[{}^{a}S\right]$ and $\left[{}^{f}S\right]$ are the compliance matrix of the arm and the forearm, $[U]$ is the transition matrix between the arm coordinate system at point C and the forearm coordinate system at point D and is given by (5) [19]:

$$[U]=\begin{bmatrix} L & -L\sin\hat{C} & L\cos\hat{C} \\ 0 & -\cos\hat{C} & \sin\hat{C} \\ 0 & -\sin\hat{C} & \cos\hat{C} \end{bmatrix} \tag{5}$$

Matrix $[S]$ established in equation (4) is reversible. The matrix $[S]^{-1}$ can be denoted:

$$[S]^{-1}=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}. \tag{6}$$

The stiffness matrix of the leg is given by:

$$[K]=\begin{bmatrix}0&0&0&0&0&0\\0&k_{11}&0&k_{12}&0&k_{13}\\0&0&0&0&0&0\\0&k_{21}&0&k_{22}&0&k_{23}\\0&0&0&0&0&0\\0&k_{31}&0&k_{32}&0&k_{33}\end{bmatrix} \tag{7}$$

The stiffness matrix is singular. This singularity corresponds to passive joints which are very common in parallel robots. The supplemental zeros are characteristic of the remaining DOF in the isolated leg.

B. Isoglide3-T3 stiffness matrix.

The Isoglide3-T3 is made of three legs connected by the moving platform considered as infinitely rigid. The three legs are three sub-structure mounted in parallel. The Isoglide3-T3 stiffness matrix reduced to its characteristics point P_{T3} is the sum of the three leg stiffness matrices reduced to P_{T3}.

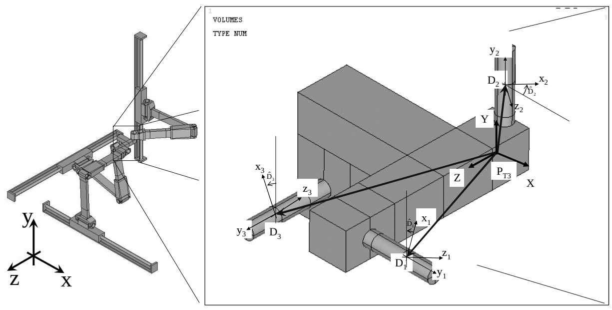

Fig.3: Moving platform of the Isoglide3-T3

Then, the Isoglide3-T3 stiffness matrix is:

$$\left[{}^{T3}K\right]=\sum_{i=1}^{3}[B_i]^T[P_i]^T\left[{}^{i}K\right][P_i][B_i] \tag{8}$$

with:

$$[P_i]=\begin{bmatrix}R_i&0_3\\0_3&R_i\end{bmatrix} \tag{9}$$

$$[B_i]=\begin{bmatrix}I_3&\left[\overrightarrow{PD_i}^{\wedge}\right]\\0_3&I_3\end{bmatrix} \tag{10}$$

where:

- $[{}^{i}K]$ is the stiffness matrix of L_i calculated in (7).
- $[R_i]$ is the transition matrix between the local coordinate system of L_i and the global coordinate system
- $\left[\overrightarrow{PD_i}^{\wedge}\right]$ is the pre-cross product matrix of the vector $\overrightarrow{PD_i}$.
- I_3 is the 3x3 identity matrix.
- 0_3 is the 3x3 null matrix.

Equation (8) gives the stiffness matrix of the Isoglide3-T3 reduced to its characteristic point. This matrix allows calculating loads generated by the Isoglide3-T3 when it is subjected to an imposed displacement. On the other hand, the inverse of $\left[{}^{T3}K\right]$ gives the displacements of P_{T3} due to the elastic deformations of the Isoglide3-T3 when it is subjected to an external load at P_{T3}.

C. Isoglide4-T3R1 stiffness matrix.

The stiffness matrix of the Isoglide3-T3 can be used in the calculation of the Isoglide4-T3R1 stiffness matrix due to its modular design. In fact, the Isoglide3-T3 and the fourth leg L_4 can be considered as two sub-structures connected in parallel by the Isoglide4-T3R1 moving platform considered infinitely rigid. The Isoglide4-T3R1 stiffness matrix is the sum of the Isoglide3-T3 and the L_4 stiffness matrices.

Legs in the Isoglide3-T3 are connected to the moving platform by revolute joints and their stiffness matrices calculated by eq (7) are relative to a revolute joint at D. In Isoglide4-T3R1, L_4 is connected to the moving platform by a universal joint. The Isoglide3-T3 is connected to the platform by a revolute joint that is not yet taken into account in the stiffness matrix $[{}^{T3}K]$ (8). To calculate the stiffness matrix of the Isoglide4-T3R1 a twist should be applied at P, induced displacement at P_{T3} and D_4 should be calculated and consequently loads generated. The revolute joint at P_{T3}, cannot transmit any torque around the z-axis between the Isoglide3-T3 and the moving platform. Consequently, the Isoglide3-T3 rotation around the z-axis is not constrained by the moving platform at P_{T3}, it is imposed by the static equilibrium. In fact, if:

$$\left[{}^{T3}K\right]=\left[{}^{T3}k_{ij}\right]\ (i=1..6,\ j=1..6) \tag{11}$$

is the stiffness matrix of the Isoglide3-T3 calculated in (8). The fact that the torque around the z axis is null gives the stiffness matrix $[K_{T3}]$ of the leg formed by the Isoglide3-T3 [19]:

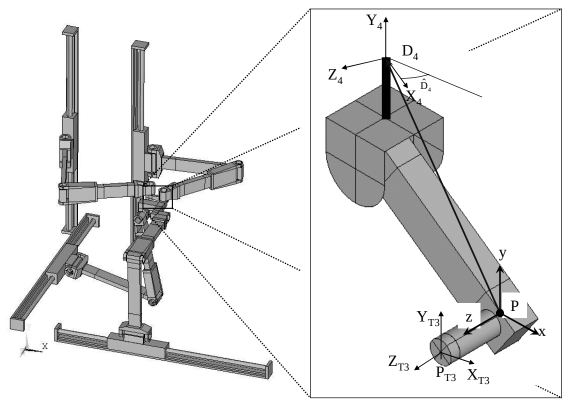

Fig.4: Moving platform of the Isoglide4-T3R1

$$\left[K_{T3}\right]=\left(I_6-\frac{1}{{}^{T3}k_{66}}\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\begin{bmatrix}{}^{T3}k_{61}\\{}^{T3}k_{62}\\{}^{T3}k_{63}\\{}^{T3}k_{64}\\{}^{T3}k_{65}\\{}^{T3}k_{66}\end{bmatrix}^T\right). \qquad (12)$$

On the other hand, L_4 is connected to the moving platform by a universal joint. Torques around the z-axis and the y-axis in the universal joint are zero. Geometric conditions impose that rotational deformation of L_4 around the x-axis is equal to the imposed rotation to P around the x-axis because the moving platform is considered as infinitely rigid. The problem unknowns are loads in L_4 and its deformations at D_4. The static equilibrium constraints give 3 equations and geometry constraints give 3 equations Finally there is a linear set, made by 6 equations and 6 unknowns, from which it will be possible to get a linear relation between twist t_P imposed to P and wrench w_4 generated by L_4. This relation can be expressed in matrix form by:

$$\left[w_4\right]=\left[{}^{4e}K\right]\left[t_P\right] \qquad (13)$$

$\left[{}^{4e}K\right]$ is equivalent to a stiffness matrix for L_4 taking in consideration the universal joint. Finally the stiffness matrix of the Isoglide4-T3R1 is:

$$\left[{}^{T3R1}K\right]=\left[B_{T3}\right]^T\left[K_{T3}\right]\left[B_{T3}\right]+\left[{}^{4e}K\right], \qquad (14)$$

where $[B_{T3}]$ is the same as $[B_i]$ corresponding to the point P_{T3}.

Equation (14) gives the stiffness matrix of the Isoglide3-T3R1 at each point of its workspace. Based on this equation, stiffness maps of the Isoglide4-T3R1 can be drawn, such as the deformation of the Isoglide4-T3R1 under an external load can be estimated.

V. Application

A real prototype of the Isoglide4-T3R1 was built at the mechanical engineering research group with the collaboration of the LASMEA in Clermont-Ferrand. Calculation with the FEM [19] gives:

$$\left[{}^{a}S\right]=\begin{bmatrix}3.20\ 10^{-4} & 0 & 7.42\ 10^{-7}\\ 0 & 1.13\ 10^{-8} & 0\\ 7.42\ 10^{-7} & 0 & 2.44\ 10^{-9}\end{bmatrix} \qquad (15)$$

and:

$$\left[{}^{f}S\right]=\begin{bmatrix}7.15\ 10^{-4} & -2.66\ 10^{-7} & 1.24\ 10^{-6}\\ -2.66\ 10^{-6} & 5.61\ 10^{-8} & 1.15\ 10^{-9}\\ 1.24\ 10^{-6} & 1.15\ 10^{-9} & 5.25\ 10^{-9}\end{bmatrix}. \qquad (16)$$

as compliance matrices for the arm and the forearm

In $[{}^{a}S]$ and $[{}^{f}S]$ length are measured in mm, forces in N and angles in radian.

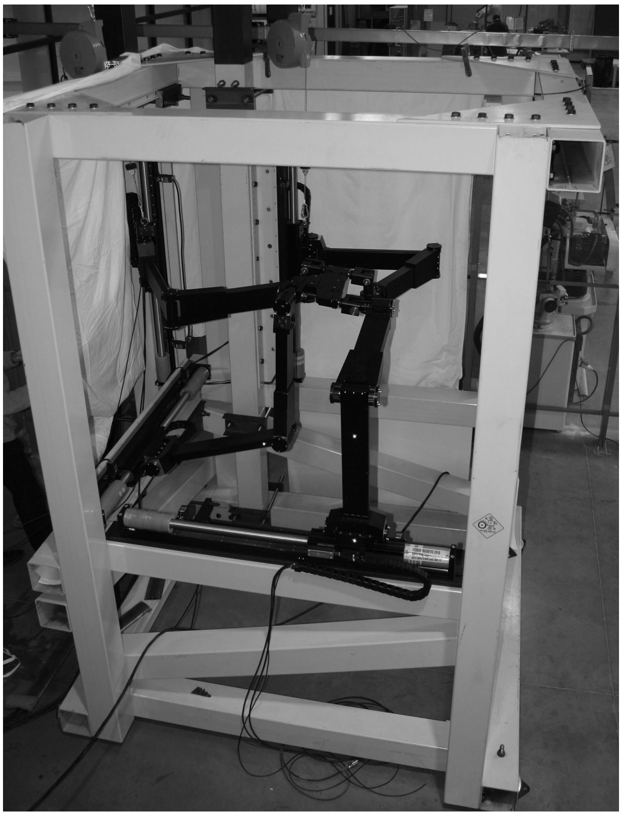

Fig.5: real prototype of the Isoglide4-T3R1

The injection of $\left[{}^{a}S\right]$ and $\left[{}^{f}S\right]$in equations (8) and (14) allows getting a semi-analytical model for the stiffness matrix of the Isoglide3-T3 and the Isoglide4-T3R1, from which stiffness maps can be deduced (Fig. 6 and Fig. 7). From the stiffness maps, it is possible to remark that terms k_{11} and k_{33} do not change a lot between the Isoglide3-T3 and the Isoglide4-T3R1. The term k_{22} of the Isoglide4-T3R1 is approximately doubled with respect to Isoglide3-T3 with a smaller relative difference between its maximum and minimum values. In both configurations, the term k_{11} is the smallest among the first three diagonal components of the stiffness matrix. On the other hand, components corresponding to the rotation and the moment around z-axis are smaller in Isoglide4-T3R1. It can be noticed that $\left[K_{56}\right]$ is smaller in the Isoglide4-T3R1. This means that the moment around y-axis loading the Isoglide4-T3R1 during a pure rotational deformation of the moving platform around the z-axis, can be neglected. This could be explained by the release of one degree of mobility.

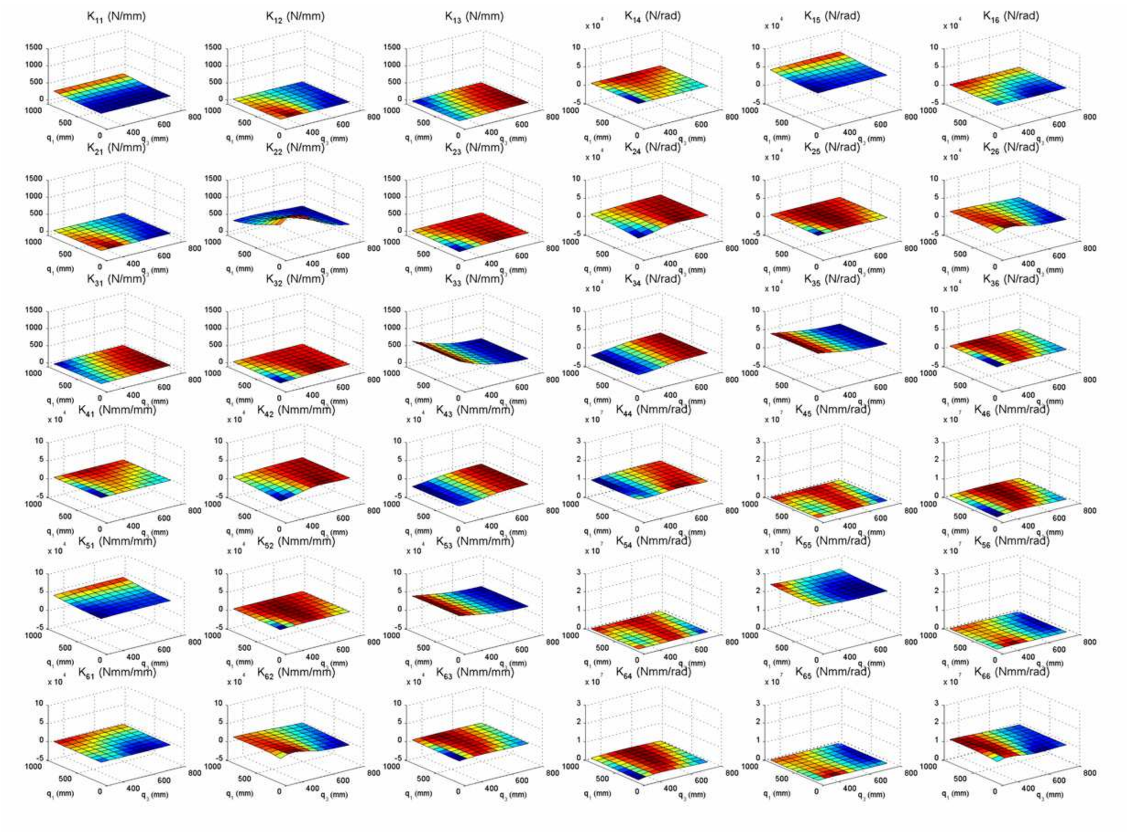

Fig.6: Stiffness maps of the Isoglide3-T3

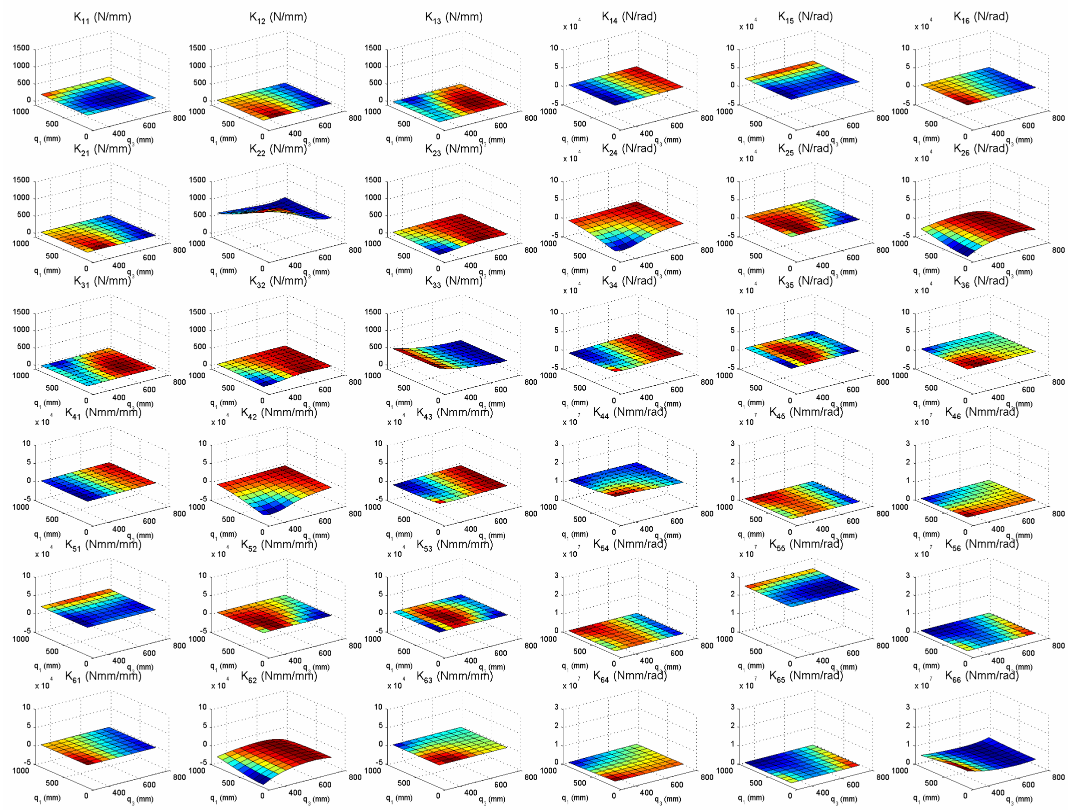

Fig.7: Stiffness maps of the Isoglide4-T3R1

VI. Conclusions and prospects

In this paper, a stiffness study for two parallel robots with the sub-structuring principle was presented. The modular aspect of the robot was used to calculate the stiffness matrix of a leg. This stiffness matrix was then used to calculate the stiffness matrix of the Isoglide3-T3 and both stiffness matrices were used to calculate that of the Isoglide4-T3R1. A numerical application was carried out. From this numerical application it was possible to highlight the effects of the fourth leg such as that of the fourth degree of freedom. Our semi-analytical model based on sub-structuring allows us to reduce the computation time at about 5 seconds with respect to corresponding time of 60 hours necessary for a classical FEM calculation without sub-structuring as reported in [16]. This is the strongest point of this methodology.

In this paper, the manufacturing defects in the robot structure and the effects of the industrial tolerances were ignored. In reality, they would induce parallelism defects between the revolute joint axes. Moreover, during loops closure pre-stress are introduced in the robot structure. The analysis of these prestress effects on the robot stiffness requires a non-linear analysis. This problem will be the subject of a next paper.

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References:

[1] Merlet J.-P., "Parallel robots", *Dordrecht: Kluwer* Academic 2000

[2] Gogu G "Structural synthesis of fully-isotropic translational parallel robots via theory of linear transformations", European *Journal of Mechanics, A/Solids*, vol. 23, pp. 1021-1039, 2004.

[3] Gogu G., "Fully isotropic hexapods", *in: Lenarcie, J and Roth, B.* Advances In Robot Kinematics,Springer, Dordrecht / Boston / London, 2006, p. 323-330

[4] Gogu G. "Structural synthesis of parallel robotic manipulators with decoupled motions", *Internal Report ROBEA MAX-CNRS*, 2002.

[5] Bouzgarrou B.C., Fauroux J.C., Gogu G., Herrah Y, Rigidity analysis of T3R1 parallel robot with uncoupled kinematics, *International symposium of robotics*, Paris, 2004

[6] Carbone G., Lim H, Takanishi A., Ceccearelli M., Stiffness analysis of biped humanoid robot WABIAN-RIV, *Mechanism and Machine Theory*, 41, 2006, 17-40

[7] Chakarov D., Optimization synthesis of parallel manipulators with desired stiffness, *Journal of Theoretical and Applied Mechanics* 28 (4) (1998)

[8] Chanal H., Duc E., Ray P., A study of machine tool structure on machining processes, *International Journal of Machine Tools & Manufacture*, 46, 2006, 98-106

[9] Company O., Pierrot F., Modelling and design issues of a 3-axis parallel machine-tool, *Mechanism and Machine Theory*, 37, 2002, 1325-1345

[10] Corradini Ch., Fauroux J.C., Krut S., Company O., Evaluation of a 4-Degree of Freedom Parallel Manipulator stiffness, *11th World Congress in Mechanism and Machine Science*, August 18-21, 2003, Tianjin, China, China Machinary press, edited by Tian Huang

[11] Gosselin C., Stiffness mapping for parallel manipulators, *IEEE Transactions on Robotics and Automation* 6 (3) (1990) 377–382.

[12] Gosselin C., Zhang D, Stiffness analysis of parallel mechanisms using a lumped model, *International Journal of Robotics and Automation* 17 (1) (2002) 17–27

[13] Rafael A., Goizalde A.., Victria H., Hernandez A., A procedure based on finite element for the solution of nonlinear problems in the kinematic analysis of mechanisms, *Finite Element in Analysis and Design* 22, 1996, 305-327

[14] Rivin E.I., *Mechanical Design of Robots*, McGraw-Hill, New York, 1988

[15] Wu Z., Rizk R., Fauroux J.C., Gogu G., numerical simulation of parallel robots with decoupled motions and complex structure in a modular design approach, *IDMME, May 17-19, 2006*, Grenoble France, acts on CD-ROM

[16] Rizk R., Fauroux, J.C., Mumteanu M., Gogu G. " A COMPARATIVE STIFNESS ANALYSIS OF A RECONFIGURABLE PARALLEL MACHINE WITH THREE OR FOUR DEGREES OF MOBILITY", *Journal of machine engineering*, Vol. 6 N° 2, 2006, P45-55

[17] Zhang D.G, Angeles J., impact dynamics of flexible-joint robots, *Computers & Structures*, 83, 2005, 25-33

[18] Gratsh T., Bathe K.-J., A posteriori error estimation techniques in practical finite element analysis, *Computers and Structures*, 83 , 2005, 235-265

[19] Rizk R., Study of the accuracy and the stiffness of parallel robots with decoupled motions, *Ph.D thesis* Clermont Ferrand, 2006