Geometric identification of a car suspension mechanism based on part displacement analysis

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Abstract—In this paper a new method is presented for the geometric model definition of a car suspension mechanism without disassembly of the suspension. This method is based on the coordinate measurement of reference points on each part in the mechanism for various load cases. These measurements allow the computation of part displacements in the operating mechanism. Using this information, the authors propose an identification method for each of the common joint that can be found in car suspension mechanism. A numerical validation of these methods is led on a pseudo McPherson suspension.

Keywords: Car suspension, geometric parameters, joint location, identification.

I. Introduction

To understand the behaviour of an existing mechanism and to study potential design improvement, the definition of an elastokinematic model is an essential step. An immediate way to achieve this task is to measure the properties of each component of the mechanism separately. But in some cases it is not possible to disassemble the mechanism and the model has to be defined from observations made on the operating mechanism.

Previous methods proposed for parameter identification of car suspension are based on the analysis of wheel motion under quasi-static load cases. Geometric and stiffness parameters can be obtained using optimization routines on an elastokinematic model of the suspension [1]. But we know that these techniques imply a high number of simulations, and advanced numerical computing techniques. This paper proposes an identification method that is based on the observation of part displacement in the suspension mechanism when various loads are applied on the wheel.

II. Part displacement observation

In order to compute part displacement on the assembled suspension, four reference points, also called ‘marks’ are made on each part of the mechanism. These marks are small conic holes located on parts to ensure an easy coordinate measurement on the assembled suspension using a portable CMM equipped with a spherical probe. We know that the minimum number of points required to compute the location and orientation of a solid part in space is three. However, four marks have been used to enhance reliability in the event of measurement error. No precise location is required for the mark, provided they are sufficiently spaced from each other and accessible for coordinate measurement.

The first measurement operation is done for a standard load case of the vehicle: two persons of 75 Kg at the front places and full tank. This measurement must be made in a coordinate system similar to the one used for suspension modelling. This global coordinate system Rg is defined conventionally with a vertical Z axis, the X axis in the human body [3], while teleoperation data can be used to identify constraints in remote environments [4]. But none of these solutions can be applied to car suspension because of part accessibility in the mechanism and reduced available space under the vehicle. It is then necessary to define an experimental protocol that meets the precision and robustness requirements of this application.

In a prior work [5], the authors have presented a method to compute part location on an assembled suspension mechanism using a portable coordinate measuring machine (CMM), but this method requires to disassemble the mechanism. The next section will show that a similar experimental protocol can be used to determine, without disassembly, part displacements when various load cases are applied to the vehicle. The third section explains how these observed displacement can be used to compute the location of spherical joints and the location and orientation of revolute joints or other complex joints specific to automobile suspension. These computational methods are tested numerically in the fourth section using an elastokinematic model of a pseudo McPherson front suspension.
forward oriented longitudinal direction of the vehicle, and the origin centred between the two tyre contact patches.

Coordinates of each mark are measured and results are saved in an IGES file. The analysis of this file defines a set of four position vectors for each part: 

$$\begin{bmatrix}
0_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} = 
\begin{bmatrix}
x_{s-a} \\
y_{s-a} \\
z_{s-a}
\end{bmatrix}_{s-a}$$

(1)

In this expression, the superscript 0 is given for the reference load case, subscript s indicates the solid part number and subscript a indicates the mark number.

This initial state of the mechanism is represented in figure 1-a.

To compute part displacement it is necessary to define a local coordinate system Rs on each part s. To simplify forthcoming computations, all Rs are initially super imposed to Rg. As a consequence there is an identity between mark coordinates expressed in Rs and in Rg:

$$\begin{bmatrix}
0_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} = 
\begin{bmatrix}
1_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g}$$

(2)

To analyse the part motions in the mechanism, mark coordinates have to be measured for various states of the suspension. Consequently the load of the vehicle has to be modified and, in the case of a front axle, the steering suspension. Consequently the load of the vehicle has to be modified and, in the case of a front axle, the steering mechanism used.

For each load case l, comprised between 1 and \(l_{max}\), each mark position is measured:

$$\begin{bmatrix}
0_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} = 
\begin{bmatrix}
x_{s-a} \\
y_{s-a} \\
z_{s-a}
\end{bmatrix}_{l_{max}}$$

(3)

where the superscript l indicates the measured load case.

For each load case l, comprised between 1 and \(l_{max}\), each mark position is measured:

$$\begin{bmatrix}
0_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} = 
\begin{bmatrix}
1_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g}$$

(4)

In this expression, \(t_{Rs/Rg}\) represents the translation vector of Rs relatively to Rg, and \(R_{Rs/Rg}\) is a rotation matrix that represents the orientation of Rs relatively to Rg. The computation of these translation and rotation is known as the absolute orientation problem [6] and consists in minimizing the criterion \(\varepsilon\) with:

$$\varepsilon = \sum_{l=1}^{l_{max}} \left\| \begin{bmatrix}
0_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} - \begin{bmatrix}
1_{s-a} \\
p_{s-a}
\end{bmatrix}_{R_g} \right\|^2.$$  

(5)

Four algorithms are presented in [6] to solve this problem but we chose to use the direct solution developed by Arun et al. [7].

In order to compute the location of a joint linking two parts, S1 and S2, it is necessary to express the displacement of one part relatively to another. The displacement of part S2 relative to part S1 is represented by the translation \(t_{2/1}\) and the rotation \(R_{2/1}\). These two terms describe the position of \(R_{2/1}\) relative to \(R_{1/2}\) and are computed with:

$$\begin{bmatrix}
t_{2/1} \\
R_{2/1}
\end{bmatrix}_{R_{2/1}} = 
\begin{bmatrix}
t_{R_{2/1}/R_{12}} \\
R_{R_{2/1}/R_{12}}
\end{bmatrix}_{R_{12}} - 
\begin{bmatrix}
t_{R_{12}/R_{2/1}} \\
R_{R_{12}/R_{2/1}}
\end{bmatrix}_{R_{2/1}}$$

(6)

At the end of the measuring operations, a number of \(l_{max}\) translation-rotation transformations are known for every couple of parts in the mechanism. The next section will explain how to use these relative displacements to compute joint positions and orientations.

III. Identification of joint location and orientation

A. Spherical joint

If a spherical joint connects part S1 to part S2, then the centre of the joint is the only point of S2 that cannot move relatively to S1. This relative displacement is represented in figure 2. Consequently, the centre of the spherical joint is the point that is not modified by any of the \(l_{max}\) translation-rotation transformations:

$$c_{Sph} = \begin{bmatrix}
t_{2/1} \\
R_{2/1}
\end{bmatrix}_{l_{max}}$$

(7)

with \(c_{Sph}\) being the position vector of the centre of the spherical joint.
\[ \mathbf{c}_{\text{Sph}} = [x_{\text{Sph}} \ y_{\text{Sph}} \ z_{\text{Sph}}]^T. \] (8)

Fig. 2. Mobility of two parts connected with a spherical joint

It is possible to identify the location vector \( \mathbf{c}_{\text{Sph}} \) using a set of linear equations. Firstly, equation (7) is modified to have a single unknown vector:

\[ (i \mathbf{R}_{2/1} - \mathbf{I}) \mathbf{c}_{\text{Sph}} = -i \mathbf{t}_{2/1} \ \forall \ l \in [1 \ldots l_{\text{max}}], \] (9)

where \( \mathbf{I} \) is the 3x3 identity matrix. Then all the load cases are considered making a concatenation of the different translation-rotation transformations:

\[
\begin{bmatrix}
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
l_{\text{max}} \mathbf{R}_{2/1} - \mathbf{I}
\end{bmatrix}
\begin{bmatrix}
x_{\text{Sph}} \\
y_{\text{Sph}} \\
z_{\text{Sph}} \\
l_{\text{max}} \mathbf{t}_{2/1}
\end{bmatrix} = 
\begin{bmatrix}
-i \mathbf{t}_{2/1} \\
-i \mathbf{t}_{2/1} \\
-i \mathbf{t}_{2/1} \\
-i_{l_{\text{max}}} \mathbf{t}_{2/1}
\end{bmatrix}. \] (10)

Equation (10) is a system of \( 3l_{\text{max}} \) equations for 3 unknown parameters. This is an unconstrained linear optimisation problem that can be solved with direct methods [8].

**B. Revolute joint**

If a revolute joint connects part \( S1 \) to part \( S2 \), the identification must define the position and orientation of the axis of this joint on the parts. To describe this axis, we chose to define a point \( \mathbf{C}_{\text{Rev}} \) and a unit vector \( \mathbf{u}_{\text{Rev}} \) as represented in figure 3.

In the case of a perfect revolute joint, all the points of part \( S2 \) located on the axis are immobile on part \( S1 \). Consequently, any point of the axis should verify:

\[ \mathbf{c}_{\text{Rev}} = \mathbf{t}_{2/1} + (i \mathbf{R}_{2/1} \mathbf{c}_{\text{Rev}}) \ \forall \ l \in [1 \ldots l_{\text{max}}]. \] (11)

This equation is identical to equation (7) and in a similar way to spherical joint we can define a system of \( 3l_{\text{max}} \) scalar equations as presented in equation (10). The matrix associated to this system will be called \( \mathbf{M}_{2/1} \) with:

\[
\mathbf{M}_{2/1} = 
\begin{bmatrix}
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
\mathbf{I} \mathbf{R}_{2/1} - \mathbf{I} \\
l_{\text{max}} \mathbf{R}_{2/1} - \mathbf{I}
\end{bmatrix}
\begin{bmatrix}
x_{\text{Rev}} \\
y_{\text{Rev}} \\
z_{\text{Rev}} \\
l_{\text{max}} \mathbf{t}_{2/1}
\end{bmatrix}. \] (12)

Ideally there is infinity of solutions for this system of equations. Then the matrix \( \mathbf{M}_{2/1} \) should be singular and have a kernel of dimension one. This kernel can be spanned by a unit vector \( \mathbf{u}_{\text{Rev}} \) that indicates the orientation of revolute joint axis.

\[ \ker(\mathbf{M}_{2/1}) = \{ k \mathbf{u}_{\text{Rev}} \}, k \in \mathbb{R} \Rightarrow \mathbf{M}_{2/1} \mathbf{u}_{\text{Rev}} = \mathbf{0}. \] (13)

If \( \mathbf{c}_{\text{Rev}} \) is a particular solution to equation (11) then we can verify that any sum of \( \mathbf{c}_{\text{Rev}} \) and a multiple of \( \mathbf{u}_{\text{Rev}} \) is a solution:

\[ (i \mathbf{R}_{2/1} - \mathbf{I}) (\mathbf{c}_{\text{Rev}} + k \mathbf{u}_{\text{Rev}}) = -i \mathbf{t}_{2/1} \ \forall \ k \in \mathbb{R}. \] (14)

But in fact, because of measurement uncertainties and real behaviour of joints and solid parts, \( \mathbf{M}_{2/1} \) is not singular. It is then possible to find a single point \( \mathbf{c}_{\text{Rev}} \) that minimizes the least square criterion for equation (11). To compute the axis orientation, we use the singular value decomposition of \( \mathbf{M}_{2/1} \):

\[
\mathbf{M}_{2/1} = \mathbf{U} \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{bmatrix} \mathbf{R}_{\text{Rev}}^T.
\] (15)

In this expression, \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the singular values of \( \mathbf{M}_{2/1} \), and \( \mathbf{R}_{\text{Rev}} \) is the 3x3 rotation matrix that indicates the singular directions. If \( \mathbf{M}_{2/1} \) is near to be singular then a singular value should be far smaller than the two others. The singular direction corresponding to this singular value is then the best estimation of the rotation axis of the revolute joint. If \( \lambda_3 \) is the smallest singular value, \( \mathbf{u}_{\text{Rev}} \) is given by:
In a car suspension the location of the spherical joint connecting the tie rod to the steering rack is an important parameter for the vehicle behaviour. But it is not possible to observe experimentally the displacement of the steering rack. This part is generally protected and hidden by other parts. That is why the location of this spherical joint $c_{\text{sph}}$ and the translation direction $u$ should be identified from the computed motion of the tie rod relatively to the chassis.

Figure 2 represent this system. Figure 4-a shows the mechanism at the reference load case. The location $\mathbf{c}_{\text{sph}}$ and the direction $\mathbf{u}$ should be defined in the global coordinate system $R_g$. Figure 4-b represents various possible positions of the tie rod when the steering mechanism is used. It can be seen that the centre of the spherical joint $c_{\text{sph}}$ moves along the direction $\mathbf{u}$.

The steering rack is generally linked to the chassis with a translational joint. As a consequence the centre of the spherical joint $c_{\text{sph}}$ can move along a straight line when the steering mechanism is used. If we compute the inertial tensor of the point cloud constituted by all the $\mathbf{c}_{\text{sph}}$, the inertial moment around the direction $\mathbf{u}$ should be null. The successive positions of $c_{\text{sph}}$ on the chassis $l_i$ are given by:

$$l_i^{c_{\text{sph}}} = l_1^{c_{\text{sph}}} + R_{2/1} \mathbf{c}_{\text{sph}} + v \mathbf{u}$$

The gravity centre $G$ of this point cloud is given by:

$$G = [x_G \ y_G \ z_G]^T = \frac{1}{l_{\text{max}}} \sum_{i=1}^{l_{\text{max}}} \mathbf{c}_{\text{sph}} - G$$

It is then possible to compute the inertial tensor $\tau_G$ of this point cloud at the gravity centre $G$ using the following expression:

$$\tau_G = \sum_{i=1}^{l_{\text{max}}} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

From this inertia tensor we can compute eigenvalues and eigenvectors:

$$\tau_G = [u \ v \ w] = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

In this expression, $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are eigen vectors that indicate the principal axes of inertia. $I_1$, $I_2$ and $I_3$ are the corresponding eigenvalues that represents the principal inertia moments. If all the $\mathbf{c}_{\text{sph}}$ points are on a straight line, then one momentum should be null and the corresponding eigenvector is the translational direction of the steering rack. But in fact, because of measurement uncertainties and real behaviour of joints and solid parts, a perfectly null momentum cannot be found. However, one eigenvalue should be far smaller than the others.

To find the position of the spherical joint, the authors chose to search the point that minimizes the smallest eigenvalue of $\tau_G$. This is a non linear optimisation problem that is solved using the Nelder Mead simplex algorithm [9].

D. Elastic joints

Elastic joints, also called rubber bushings, are widely used in car suspension for vibration filtering in order to improve the vehicle comfort. A bushing is typically composed of a hollow elastomer cylinder contained between inner and outer cylindrical steel sleeves. These joints introduce difficulty into geometric identification as their behaviour depends on stiffness parameters and applied forces on the joint. Both of these data are unknown at this step of the modelling process. That is why it is necessary to approximate the position of an elastic joint using the identification method of spherical joints.
E. McPherson strut

The most common suspension type on light weight vehicle is the McPherson suspension and his derivative. In this type of suspension, the spindle plate is linked to the chassis via a cylindrical strut.

This connection leaves to the spindle plate three degrees of freedom in rotation and one degree in translation with respect to the chassis. As a consequence, it is possible to use the method presented in the precedent section to identify the orientation of the strut axis and the position of the connection between the chassis and the strut.

But in this case, the connection between the strut and the chassis is ensured by an elastic joint. Consequently, the result of the identification may not be the exact location of the joint.

IV. Numerical validation

In order to validate the method presented in this paper, a program has been written using the Matlab programming language in order to perform automatically the successive computations described above. This program is able to read a file of measurement operations in the IGES format. Then displacements are computed for each part and each load case. After that an identification algorithm is applied to each joint depending on the type of the joint to identify. Finally a result file gives a list of the joints with their computed locations and orientations.

The first validation of this program was led using data obtained from numerical simulation. An elastokinematic model of a pseudo McPherson suspension is used as reference system to identify. This model is built using the commercial simulation software ADAMS/Car. The photo presented in figure 6-a shows the original suspension mechanism. On this photo has been superimposed a representation of the marks. Each polygon indicates at his corners the location of the four marks on a part. Figure 6-b represents the corresponding elastokinematic model.

The behaviour of this suspension is simulated for a variable vertical load applied on the wheel and comprised between 1 kN and 7 kN, and a steering rack translation comprised between −40 mm and +40 mm. The coordinates of each reference point are saved for 9 different positions of the suspension model.

Based on these results, the identification method is applied and the computed joint locations are compared to the original joint locations on the elastokinematic model.

Table 1 gives for each joint of the mechanism the distance in millimetres between original joint locations and identified locations. It can be seen that the location of spherical joint are correctly identified. The location of the
spherical joint on the steering rack is also correctly identified. But the computed location of the elastic joints differs from their original location in the reference model. However, the proposed method gives a good estimation of the joint location. In the case of an elastic joint with a high stiffness allowing small deflections (Elastic joint A), the location is identified with an error of 1 mm. While in the case of a more compliant joint allowing deflection of several millimetres (Elastic joint B) the precision of the identification is of 3 mm. In the case of the McPherson strut, the identification is hardly satisfying as the location error is near 7 mm. But it should be noticed that the orientation of the strut axle is correctly identified.

Tab. 1. Results of the geometric identification

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>Joint</th>
<th>Location error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle plate</td>
<td>Front arm</td>
<td>Spherical</td>
<td>0,0</td>
</tr>
<tr>
<td>Spindle plate</td>
<td>Rear arm</td>
<td>Spherical</td>
<td>0,0</td>
</tr>
<tr>
<td>Spindle plate</td>
<td>Steering rod</td>
<td>Spherical</td>
<td>0,0</td>
</tr>
<tr>
<td>Chassis</td>
<td>Steering rod</td>
<td>spherical + translational</td>
<td>0,0</td>
</tr>
<tr>
<td>Chassis</td>
<td>Front arm</td>
<td>Elastic</td>
<td>3,2</td>
</tr>
<tr>
<td>Chassis</td>
<td>Rear arm</td>
<td>Elastic</td>
<td>1,0</td>
</tr>
<tr>
<td>Spindle plate</td>
<td>Chassis</td>
<td>Elastic + Cylindrical</td>
<td>6,9</td>
</tr>
</tbody>
</table>

V. Conclusion

This paper presents a method to compute joint location and orientation in an assembled suspension mechanism using only point coordinate measurement for various load cases of the suspension. These point coordinates allow to compute the relative displacement of each pair of solid part connected with a joint with no disassembly of the mechanism.

This relative displacement is used to compute the joint location. The computation method depends on the type of joint to identify. A different identification method is proposed for spherical, revolute and elastic joints. An identification method is also proposed for combined joints (spherical and translational) that can be seen on steering mechanism or McPherson strut.

These methods have been tested using numerical simulation on a pseudo McPherson suspension. This virtual experiment showed that the identification is correct for spherical joint. In the case of elastic joints, an estimation of the joint locations is obtained with a precision range of some millimetres depending on the joint stiffness. This precision allows the definition a realistic model of the suspension.

The experimental validation of this identification method will be the focus of a future work.

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References


APPENDIX

Notations

\( R_g \) Coordinate system global to the mechanism.
\( R_s \) Coordinate system local to the solid part \( S \).
\( \mathbf{k}_{R_0/R_g} \) Position of coordinate system \( R_s \) relative to \( R_g \) for the load case \( l \).
\( \mathbf{R}_{R_0/R_g} \) Orientation of coordinate system \( R_s \) relative to \( R_g \) for the load case \( l \).
\( \mathbf{k}_{R_2/l} \) Position of coordinate system \( R_{s2} \) relative to \( R_{s1} \) for the load case \( l \).
\( \mathbf{R}_{R_2/l} \) Orientation of coordinate system \( R_{s2} \) relative to \( R_{s1} \) for the load case \( l \).
\( \mathbf{p}_{a/s} \) Position vector of the reference point \( a \) on part \( S \), for the load case \( l \).
\( \mathbf{c}_{sp} \) Position vector of a spherical joint center
\( \mathbf{u}_{rev} \) Unitary vector of a revolute joint axis
\( \mathbf{c}_{rev} \) Position vector of a point belonging to a revolute joint axis