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USING THE SKELETON MODEL FOR PRELIMINARY GEOMETRICAL SYNTHESIS OF 3D CINEMATIC CHAINS

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1 Introduction

The first step in the design process is cinematic synthesis which aims to create a mechanism to yield a desired set of motion characteristics. A frequent design requirement is to cause an output member to rotate, oscillate, or reciprocate according to a specified function of time or of the input motion. This is called function generation. Moreover, if cinematic structures are thought of as ordered sets of constructive primitives, the resulting mechanisms can be called cinematic chains or SISO (single input single output mechanisms).

The literature provides several general computational methods for the synthesis of cinematic chains. For example, the abstract representation of geared cinematic structures was investigated with the aid of graph theory first by Buchsbaum and Freudenstein [1] and then by other authors [2-3]. More recently, researchers in computer science have proposed several methods in qualitative physics and constraint programming for the synthesis of this kind of mechanism [4-9].

Within the function generation field, the synthesis problem specification involves describing input and output motion, and a set of constraints, of which some are geometrical, e.g. the required positions and orientations of input and output members.

Most synthesis methods ignore geometrical constraints within their reasoning procedure, dealing only with structural and topological considerations. The approach suggested by Kota [6], which uses a matrix representation to model its building blocks, is one of the few which manage orientation constraints from the first synthesis process step. Note, however that it works only with the 3 orthogonal orientations (**X**,**Y**,**Z**) of the reference frame.

The first synthesis step must therefore be generally followed by the application of geometrical synthesis work to verify if the synthesized chain is able to fulfil the entire specification. For example, Chakrabarti and Bligh [8-9] start by producing a set of solution concepts for a given design problem, using kind synthesis procedures, and then continue by checking that a candidate solution concept satisfies the geometrical requirements, using a constraint propagation procedure. Working within orthogonal restrictions, they have shown that their approach makes it possible to process orientation and sense constraints; the management of position constraints is kept for the later and more detailed phases of design.

Our proposed method also consists in a multi-step solution to the cinematic chain synthesis problem:

- Step 1: from a data base including all the most common elementary mechanical blocks (EMB), a structural synthesis process produces all the global structures, as ordered sets of EMBs, which are likely to fulfil the requirements. It works in three phases: 1. Enumerating all the possible combinations, 2. Eliminating inappropriate solutions from a set of rules (some of which are geometrical, but only qualitatively), 3. Sorting the remaining solutions by order of decreasing interest. At the end of this first step, there is no guarantee that a suggested solution will be able to satisfy all the precise geometrical constraints of the specification.
- Step 2: a candidate structure having been selected, this preliminary task of geometrical synthesis is intended to find the associated 3D closed chain running from the input to the output position and respecting the main structural characteristics of each constitutive EMB. If it succeeds, an initial geometrical model of the structure is obtained.
- Step 3: this last task consists in a full synthesis which completes the one carried out previously by taking into account additional variables such as the main component dimensions (shafts and wheel diameters…) and considering conventional design criteria (contact stress, fatigue life, proportion ratios…).

This paper focuses on the second step. A model which is able to represent and position in space the main geometrical elements of any 3D speed reducer structure has already been presented in [10]. This model is based on the concept of mechanism skeleton. A CAD tool has been developed where about twenty EMBs are considered, such as different cylindrical gearings, crossed-axis helical gearing, worm gear... The main interest of this approach lies in its ability to manage accurate geometrical constraints. 3D problems with any input and output orientations may be tackled. Orientation and position requirements are considered together within an unique analytical formulation. We will show here that the skeleton technique can be extended to other types of 3D cinematic chains, especially those which also include more complex constructive primitives such as slider-crank, eccentric, rack-and-pinion, cam …

2 Problem setting

The starting point of the preliminary geometrical synthesis step is a given candidate structure, made up of a set of serially connected EMBs. The geometrical elements which constitute the specification sheet of the synthesis problem are: the parallelepipedic envelope inside which every part of the mechanism should be inscribed, the position (O_0) and orientation ($\mathbf{Z_0}$) of the input member and the position (O_S) and orientation ($\mathbf{Z_S}$) of the output member.

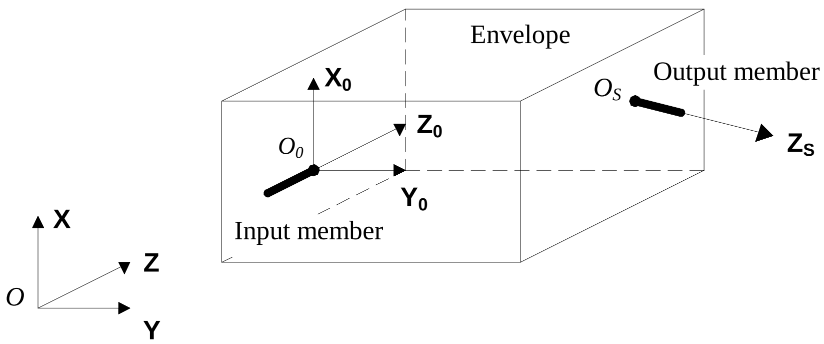
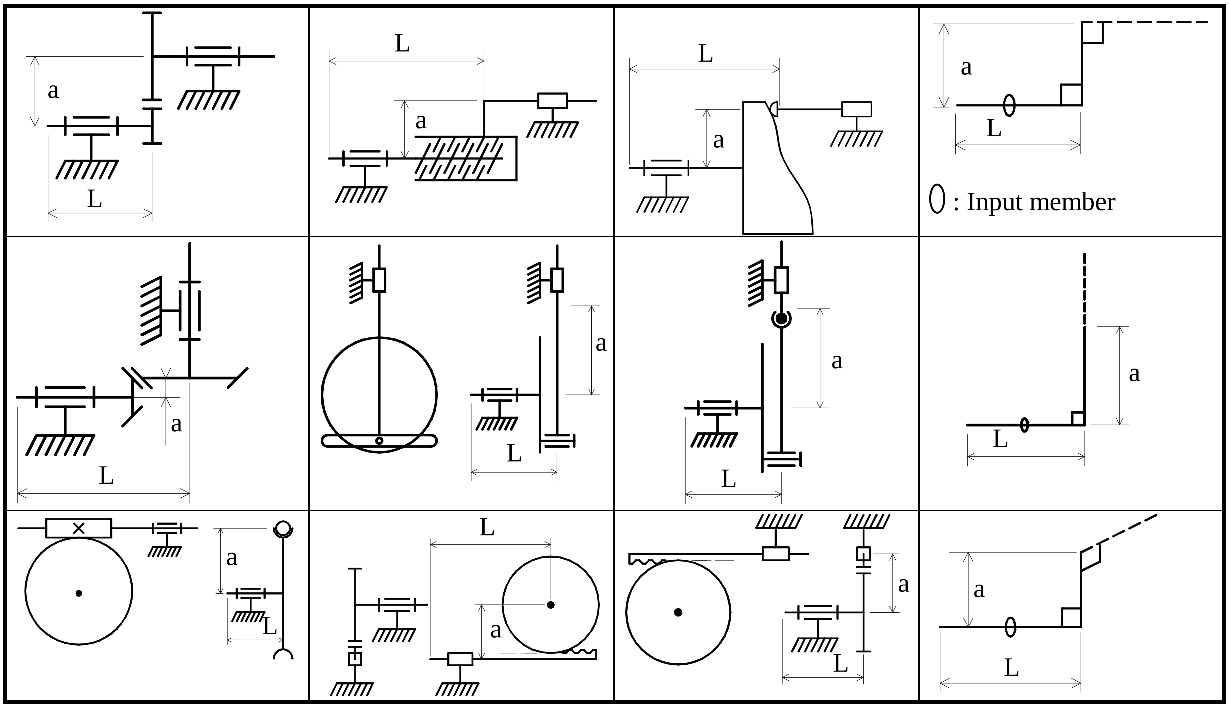


Figure 1. Specifications representation

3 Skeleton principle

The synthesis considered here is a preliminary work intended to make sure, before going any further, that the solution under study can offer a space configuration which satisfies the specifications. We have chosen to base this work on the most simplified geometrical model possible, in order to reduce calculations and thus the checking task to a minimum. We have thus eliminated all volumes from the parts and have retained only the geometric elements which play a role in the definition of the position and the orientation of the output member related to that of input. Each EMB is then represented schematically by a minimal model made up of lines which we call the "skeleton". Table 1 shows the skeleton associated with some components from our EMB database: external cylindrical gear pair, screw mechanism, cam-translating follower (line 1), bevel-gear, eccentric, slider-crank (line 2), worm-gear, rack-and-pinion (line 3). All the mechanisms which combine rotation and translation are conventionally represented in their most retracted position. Table 1 illustrates that primary mechanisms belonging to different classes can share a same skeleton. At the present time, our EMB database contains 39 mechanisms but only 10 different skeletons are necessary to model them all.

Table 1. Some EMB with associated skeletons



4 Geometrical model

Certain dimensions of a mechanism, and thus of its skeleton, may be changed without compromising the correct working order of the mechanism. At the initial design stage, the structure is not yet sized, so all the dimension and orientation parameters likely to be modified can be considered as problem variables. These variables are either lengths, or angles. These degrees of freedom can be shown schematically by prismatic and rotational joints and in this way our skeleton can be transformed into a kind of deformable structure. The mDH notations [11], well-known in the robotics field, may be used to model the space configurations of this structure. Then, the initial geometric synthesis problem can be seen as a closing chain problem, or as a robot geometrical inverse model research.

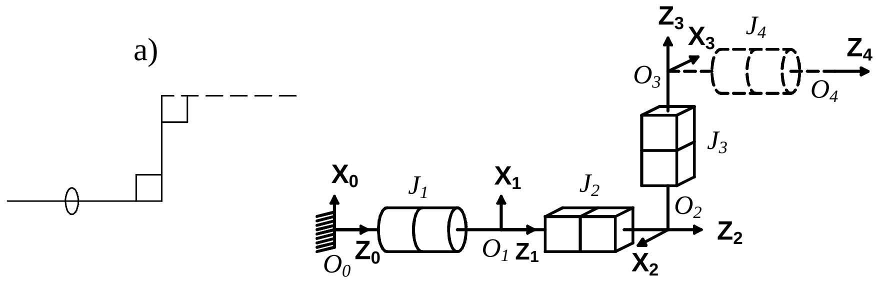
4.1 The skeleton geometrical models

The skeleton of any EMB is now considered as a series of links, the link (*j-1*) being connected to the link *j* by the joint J_j which is either a prismatic joint (length variable), or a rotational joint (angular variable). The transformation matrix allowing the change from the coordinate frame $R_{(j-1)}$, fixed with respect to the link (*j-1*), to the frame R_j is:

$$[T]_{j-1}^{j} = \begin{bmatrix} \cos\theta_j & -\sin\theta_j & 0 & d_j \\ \cos\alpha_j \sin\theta_j & \cos\alpha_j \cos\theta_j & -\sin\alpha_j & -r_j \sin\alpha_j \\ \sin\alpha_j \sin\theta_j & \sin\alpha_j \cos\theta_j & \cos\alpha_j & r_j \cos\alpha_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

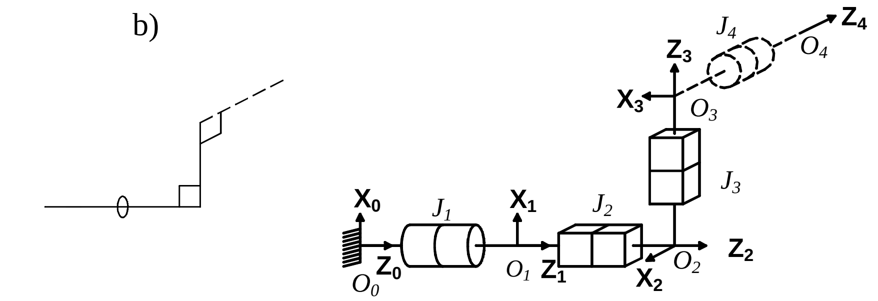
where α_j , d_j , θ_j et r_j are the 4 mDH parameters. Figure 2 gives the geometrical model of two different skeletons, the one related to mechanisms with parallel members on opposite sides (fig. 2a) and the other related to mechanisms with concurrent and perpendicular members (fig. 2b). The corresponding mDH parameters are illustrated to the right of the figure, where q_j represents the variable introduced by the joint J_j.

a)



	J_1	J_2	J_3	J_4
σ_j	0	1	1	
α_j		0	$+\Pi/2$	$+\Pi/2$
d_j		0	0	0
θ_j	q_1	$+\Pi/2$	$+\Pi$	
r_j	0	q_2	q_3	

b)



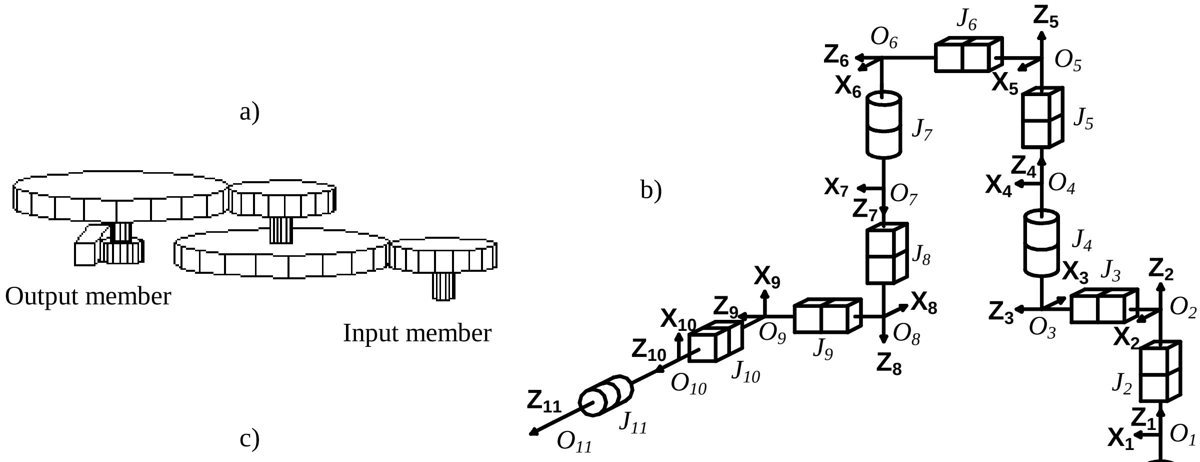
	J_1	J_2	J_3	J_4
σ_j	0	1	1	
α_j		0	$+\Pi/2$	$+\Pi/2$
d_j		0	0	0
θ_j	q_1	$+\Pi/2$	$-\Pi/2$	
r_j	0	q_2	q_3	

Figure 2. Two skeleton geometrical models

4.2 The whole mechanism geometrical model

A geometrical model is associated with each skeleton and a skeleton with each EMB, and as the entire mechanism under study is made of a set of serially connected EMBs, it is easy to obtain the geometrical model of this entire mechanism by just placing the different skeleton geometrical models of the constitutive EMB side by side. Two columns must be added at the end of the whole parameter table to represent the possible variation in the length of the last output member and the possible rotation of the structure about this output member. Note that, using the transformation matrix association, it is possible to build automatically the geometrical model of any 3D SISO mechanism.

Figure 3a shows an optical pick-up mechanism (used to move a compact disk reading head lens) made of three successive EMBs (2 gear pairs and 1 rack-and-pinion). Figure 3b gives its global geometrical model and fig. 3c the related parameters.



c)

	First Stage			Second Stage			Third Stage			Output member	
	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}
σ_j	0	1	1	0	1	1	0	1	1	1	0
α_j	0	0	$+\Pi/2$	$+\Pi/2$	0	$+\Pi/2$	$+\Pi/2$	0	$+\Pi/2$	$+\Pi/2$	0
d_j	0	0	0	0	0	0	0	0	0	0	0
θ_j	q_1	$+\Pi/2$	$+\Pi$	q_4	$+\Pi/2$	0	q_7	$+\Pi/2$	$+\Pi/2$	0	q_{11}
r_j	0	q_2	q_3	0	q_5	q_6	0	q_8	q_9	q_{10}	0

Figure 3. The whole geometrical model of a pick-up mechanism with $q_2 = q_{11} = 0$ and $q_4 = q_7 = +\Pi/2$

5 Expression of the synthesis problem

The search for a space configuration of the candidate structure is expressed as the search for q_j values which satisfy the following constraints:

- the input member being positioned according to the specification (O_0, $\mathbf{Z_0}$), the output member (O_{NJ}, $\mathbf{Z_{NJ}}$ where NJ is the total number of joints) must take the position and orientation required (O_S, $\mathbf{Z_S}$).
- the entire skeleton must lie inside the specified envelope.

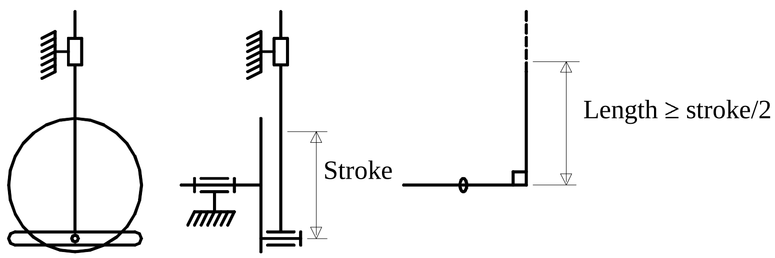


Figure 4. An example of the relationship between the specification and variable bounds

- each q_j value must remain inside its variation domain. The two limits of this domain are defined either from technological considerations (for example, the inferior limit on the number of teeth of a toothed wheel induces the minimum limit on the distance between gear shaft axes), or from an interpretation of the specification sheet. Figure 4 shows that, for an eccentric, the minimal value of a skeleton variable may be directly dependent on a stroke required in the specification.

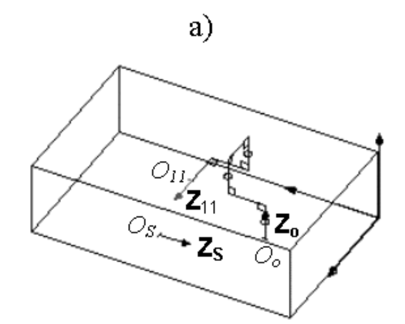
This kind of synthesis problem is generally redundant as the number of solutions is often infinite, the variable number being higher than the closure equation number. An optimisation criterion has been chosen to find a more suitable solution. As designers often prefer compact mechanisms, we have decided to minimize the overall length of the skeleton. This proves that the result thus obtained provides a good starting point for the next step of our design process (see section 1, step 3) in which technological constraints are added to control the sizes assigned to the main parts.

6 Applications

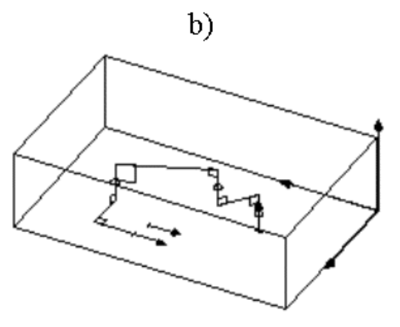
6.1 Optical pick-up mechanism.

The study of the mechanism already introduced in section 4.2 can be summarised as follows: Table 2a illustrates the initial space configuration of the structure. Default values have been given at the outset to the variables q_1 to q_{11}, leading to an incorrect orientation ($\mathbf{z_{11}}$) and position (O_{11}) of the output member. The optimisation algorithm manipulates the structure in order to satisfy first the orientation constraints (2b), then the position constraints (2c) and lastly to minimise the skeleton length (2d).

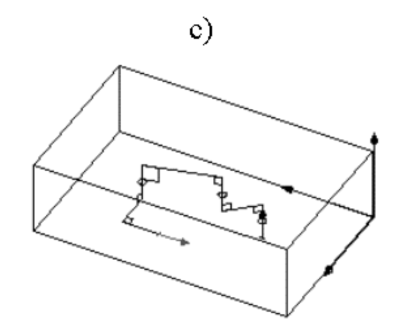
Table 2. Steps of preliminary geometrical synthesis



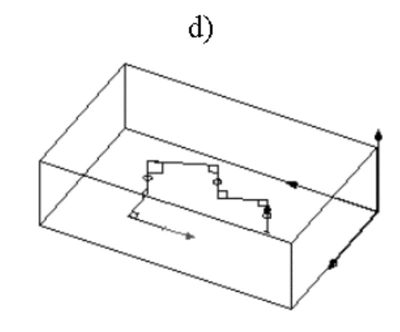
Initial step	
q_1	5 mm
q_2	0 rad
q_3	5 mm
q_4	5 mm
q_5	0 rad
q_6	5 mm
q_7	5 mm
q_8	0 rad
q_9	5 mm
q_{10}	5 mm
q_{11}	0 rad



Step 88	
q_1	5 mm
q_2	1.07 rad
q_3	5 mm
q_4	5 mm
q_5	1.16 rad
q_6	11.14 mm
q_7	5 mm
q_8	0.66 rad
q_9	5 mm
q_{10}	5 mm
q_{11}	0 rad



Step 662	
q_1	5 mm
q_2	0.76 rad
q_3	5.03 mm
q_4	5 mm
q_5	1.06 rad
q_6	10.19 mm
q_7	5 mm
q_8	0.25 rad
q_9	5.01 mm
q_{10}	5 mm
q_{11}	0 rad



Final step (1713)	
q_1	5 mm
q_2	0.45 rad
q_3	6.04 mm
q_4	5 mm
q_5	1.51 rad
q_6	8.73 mm
q_7	5 mm
q_8	0.39 rad
q_9	5 mm
q_{10}	5 mm
q_{11}	0 rad

6.2 Jigsaw mechanism

The main specifications for a jigsaw mechanism are the following (coordinates given in the general frame *Rg*): input member = rotation, O_0=(30,20,0), $\mathbf{z_0}$=(0,0,1); output member = alternate translation, stroke = 25 mm, O_S=(30,90,60) and $\mathbf{z_S}$=(0,0.87,0.5). The left side of Fig. 5 presents 3 candidate structures suggested by the first structural synthesis process. Among the numerous solutions proposed, we have deliberately kept here three combinations of two stages, the first stage varying one with the other: bevel-gear first configuration and slider-crank (fig. 5a), bevel-gear second configuration and slider-crank (fig. 5b), worm-gear and slider-crank (fig. 5c). Note that, by using only two stages, it is difficult to satisfy the specification where the input and output members are neither parallel nor orthogonal. In this case, it is useful to evaluate these solutions from a geometrical viewpoint to verify whether they are really suited to the requirements.

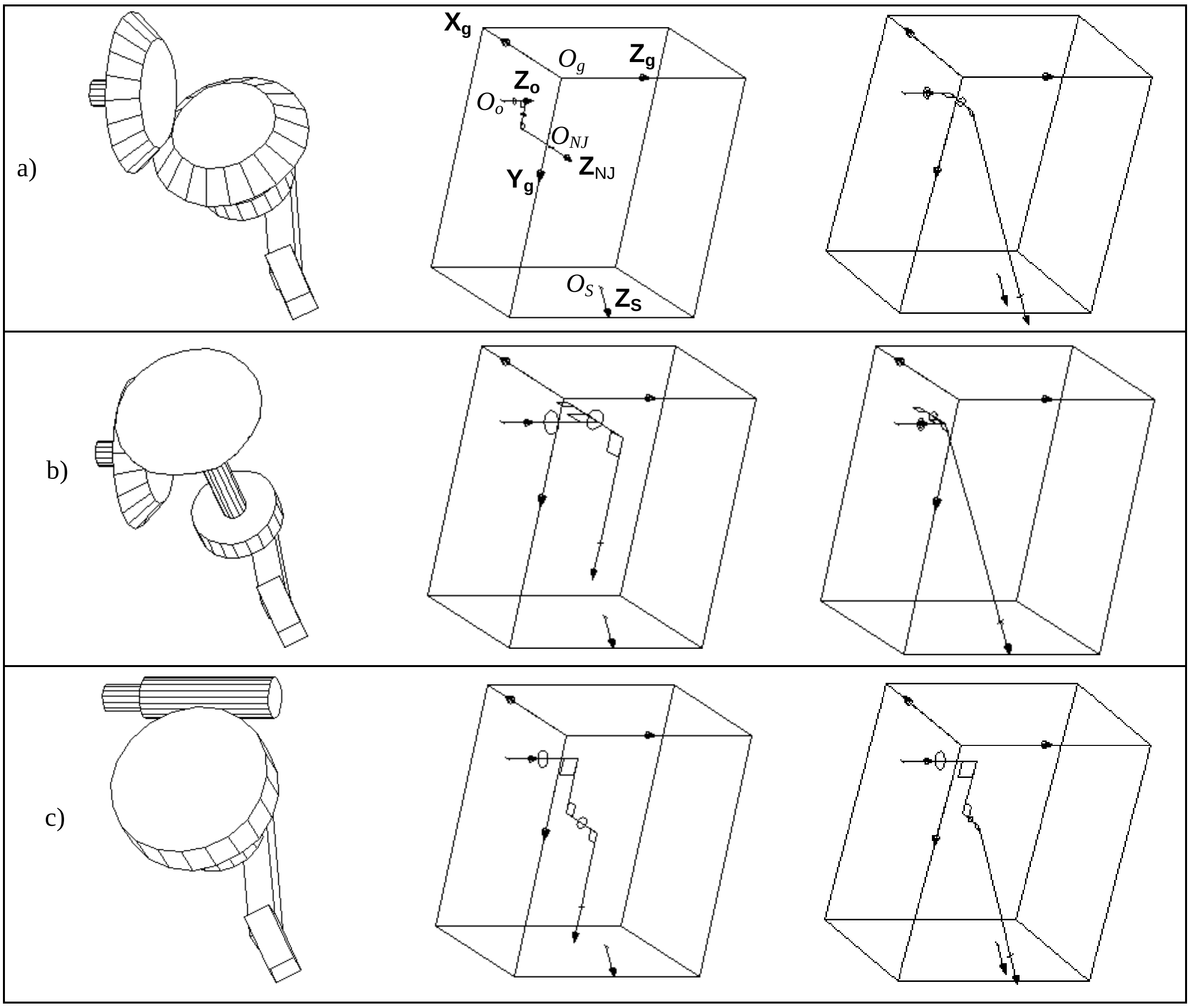


Figure 5. Preliminary geometrical synthesis of 3 structures candidate for a jigsaw mechanism

The initial and final space configuration of each combination are shown in fig. 5. They show that only the second combination is the able to satisfy both the orientation and position constraints.

7 Conclusion

The skeleton principle which consists in representing the main features of mechanism architectures by filar structures at the early stage of design is presented. The possibility of extending the skeleton concept to constructive primitives such as slider-crank, eccentric, rack-and-pinion or cam is considered. Using the well-known mDH notations, an assembly method is proposed, allowing for the automatic construction of the geometrical model of any 3D cinematic chains. This model is used within a closed chain synthesis process to help designers check whether a candidate structure can satisfy both orientation and position constraints.

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