# Using the Skeleton Method to Define a Preliminary Geometrical Model for Three-Dimensional Speed Reducers

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**Abstract.** *This paper deals with the first design phase of three-dimensional speed reducers. Concepts to help designers during the entry-level model definition are presented and, more particularly, those related to the task of geometrical synthesis. From these principles, a specific CAD tool has been built and is also described. A rough model, called a skeleton, is introduced to represent each conventional reducing stage category, thus enabling the automatic formation of entire geometrical models of speed reducers. Shaft orientations and positions are calculated from products of transformation matrices, in the same way as for spatial kinematic chain closure conventional problems. Optimization techniques are used to obtain a preliminary dimensioning of the structure. The numerical processing is achieved progressively in three steps in order to improve the final convergence and, if necessary, to enlighten further the designer on failure origins within specification data. Three examples are given to illustrate both the creation of the geometrical model and the way results are obtained.*

**Keywords:** 3D Speed reducer design; Geometrical model optimization; Mechanism synthesis; Closed-loop chain; Skeleton

## 1 . Introduction

Most common power transmission lines encountered in machines are one-degree-of-freedom mechanisms built as a series of elementary stages, such as gear, belt or chain stages, connected to each other by intermediate shafts. Their main function is to transmit a rotative movement from an input shaft to a remote output shaft whose rotation speed is often quite small in comparison with that of the input. These mechanical devices can be designed without difficulty when input and output shafts are parallel or on a same planar surface. Attention will be focused here on cases where the output shaft must take a complex spatial position with regard to the input one, such devices being called three-dimensional speed reducers (*3D.S.R.*). At least two stages with non parallel shafts, such as bevel or worm gearing, must generally be used to obtain the required output position. The design of these 3D mechanisms leads consistently to synthesis problems for the solution of which methods and assistance tools are welcome.

The purpose of *3D.S.R.* preliminary design is to establish an entry-level model of the device, which specifies the composition of the mechanism and gives starting values to positions and dimensions for all main components such as shaft axes and gearing wheels. This model, often represented by a structural scheme with some dimensional data, is built from the kinematic point of view, first considering the geometrical aspects of the problem. It constitutes the specification sheet of the next design phase, which will take into account many other aspects in a more detailed study of each device part.

The preliminary design process mentioned above may be considered as the three phase process presented in Fig. 1.

At the starting point, the main available specifications are the expected spatial positions of the input and output shafts, the required speed ratio and some other global characteristics. The first task necessarily consists in defining the nature of all the elementary reducing stages which will be serially connected to form the transmission line (Phase 1). A wide range of literature concerning the structural synthesis of mechanisms is available [1-6]. Generally based on graph representation, methods have been proposed essentially for the classification and enumeration of mechanisms according to kinematic structures. Buchsbaum and Freudenstein [2], Ravisankar and Mruthyunjaya [5] applied such methods to gear transmission and differential drives. Unfortunately, these methods do not apply to practical cases where a *3D.S.R.* architecture fulfilling given specifications is to be found. Usually, the task of structural synthesis is rather carried out using the designer's skills and knowledge. In any case, the final design choice takes the form of an ordered set of elementary stages, but without the guarantee that this choice will be appropriate nor optimal.

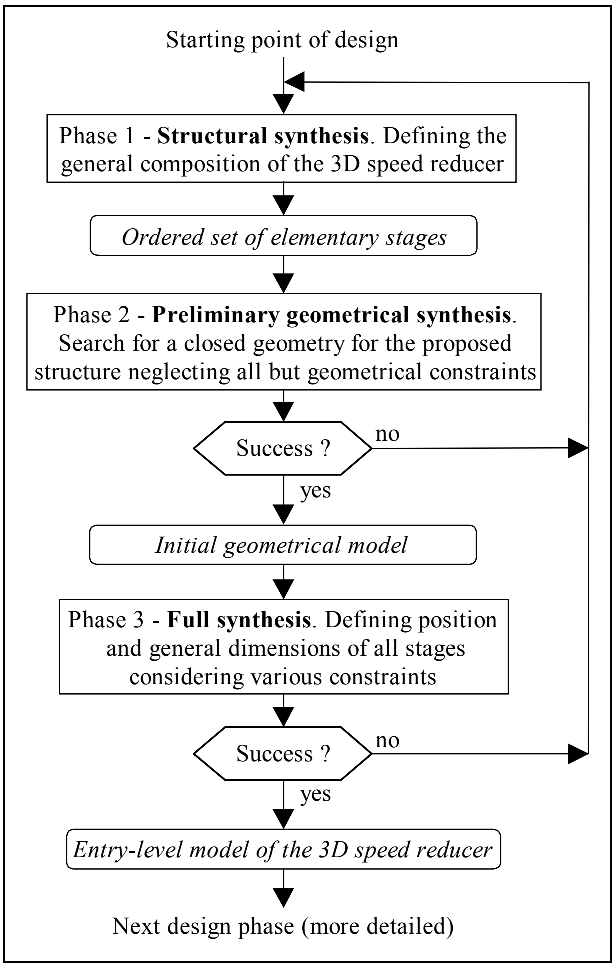



**Fig. 1.** Preliminary design process of 3D Speed Reducers

Afterwards, the definition of an entry-level model for a *3D.S.R.* consists in :

- searching for the main dimensions of structure components, i.e. searching for a closed spatial chain running from the input to the output position and respecting the structural characteristics of each selected elementary stage
- under different constraints such as :

  - obtaining the required speed ratio
  - avoiding part interference
  - using realistic wheel dimensions (taking into consideration the main conventional design criteria : contact stress, fatigue life, proportion ratios...).

The whole problem is relatively cumbersome and there is little chance of succeeding in attempts to solve it directly, especially as the existence of a closed geometry is uncertain. A more suitable way to solve this problem is to divide the resolution into two phases.

Thus, the design process first continues with a preliminary task of geometrical synthesis (Phase 2) intended merely to find a closed geometric chain. If this search fails, the previous choice (i.e. an ordered set of stages) has to be reconsidered. If it succeeds, an initial geometrical model is obtained and the process may continue using this model as a starting point. The last step is a full synthesis dealing with the complete problem (Phase 3).

In this paper, attention will only be focused on the task of preliminary geometrical synthesis and particularly on Phase 2. A software tool which has been developed in order to help the designer build *3D.S.R.* initial geometrical models is presented. The main difficulty has been to find a formulation which is general enough to allow the automatic processing of all power transmission lines, with any number and any nature of stages. This obstacle has been overcome by transforming the problem into a conventional closed spatial chain synthesis. This transformation is based on the use of filar representations of elementary stages, called stage *skeletons*, and which contain only the data necessary to build the model.

Many studies are available concerning closed spatial chain analysis or synthesis. Algebraic methods have been applied to various space bar-linkages made from rotational, spherical or prismatic joints, such as the RGGR mechanism studies presented by Shigley and Uicker [7] or by Söylemez and Freudenstein [8]. Analytical approaches, characterized by the search for mechanism closed positions from matrix loop equations, have been initiated by Denavit and Hartenberg [9], then widely used, especially in the robotics field. These latter appear to be perfectly suited to our problem because they offer the means of constructing a model by standard unit association. The relevant literature provides several methods of formulating spatial loop equations [10]. In our case of a simple closed-loop chain, Denavit and Hartenberg's notations [11] have seemed the most appropriate because of their simplicity.

Using transformation matrix association, it thus became possible to build automatically a geometrical model for any *3D.S.R*. Such a model being usually redundant, nonlinear optimization techniques have been chosen to determine the geometrical parameters that minimize the device overall dimensions.

## 2 . Classification of the reducer elementary stages

Let us begin with the presentation of the most common elementary stages which can be encountered in power transmission lines. In order to prepare the following developments, stages are classified according to their general geometrical structure. Figures 2 and 3 emphasize two main stage categories :

- *Stages with non concurrent shafts.* The most common values of the angle $\varphi$ , defined in Fig. 2 from $\vec{k}$ to $\vec{u}$ and measured in the $(\vec{i}\,,\vec{j},\vec{k}\,)$ direct ordinate system, are *0°*, *90°*, *180°* and *270°*. This category includes stages with parallel shafts either on opposite sides $(\varphi = 0°)$ or on the same side $(\varphi = 180°)$, the associated elementary mechanisms being : external/external and external/internal cylindrical gearing, belt-pulley transmission, chain transmission. Some stages with non parallel, non concurrent shafts, such as worm gearing, crossed-axis helical gearing and hypoïd gearing are also part of this class, with generally $\varphi = 90°$ or $\varphi = 270°$.
- *Stages with concurrent shafts.* When the angle $\varphi$ , defined in Fig. 3 equals *0°* or *180°*, input and output shafts are coaxial. Some epicyclic trains and specific devices such as the harmonic drive joint are concerned. Other possible devices are bevel gearing (the two usual configurations shown in Fig. 3 square with $\varphi = 90°$ but provide opposite senses of output rotation) and Cardan joint (under certain conditions).

The diagrams in the right of Fig. 2 and 3 are called the "skeletons" of the stages. These linear structures will be used later to construct the geometrical model of 3D speed reducers. Their main role is to represent the stage spatial architecture. Dimensions used to mark these architectures are only general :

- the input shaft length $L$
- the distance $a$ between input and output axis
- the angle $\varphi$ between both axes.

There is no length associated to the output shafts in order to decrease the parameter number. At this wide abstraction level, this length will be included in the input shaft length of the next stage.

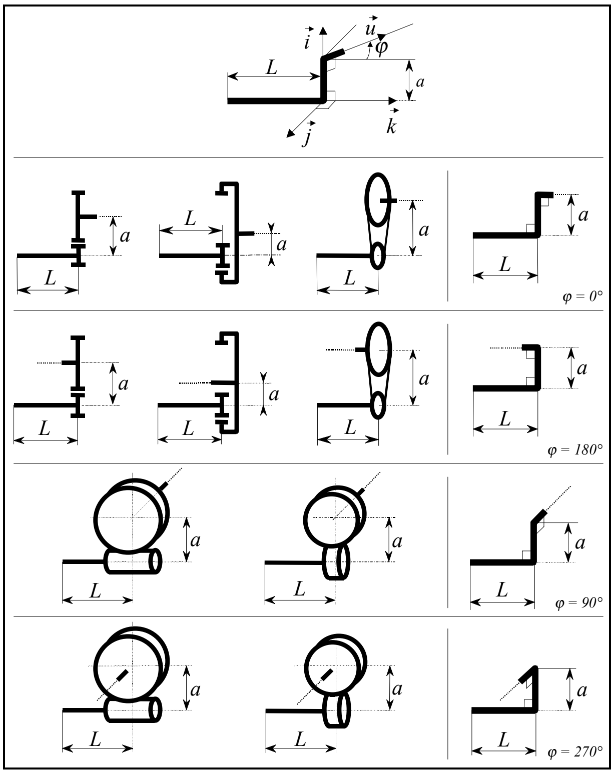



**Fig. 2.** Stages with non concurrent shafts

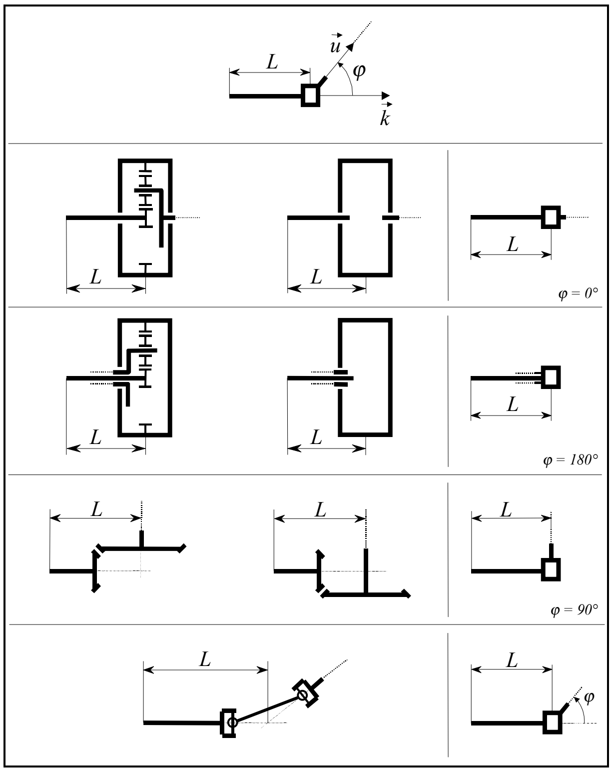



**Fig. 3.** Stages with concurrent shafts

## 3 . Problem setting

The task of preliminary geometrical synthesis requires only the following initial specifications :

- the nature of the successive stages which will constitute the power transmission line. It is assumed this preliminary choice will not necessarily lead to a feasible solution
- a coordinate system $R_0 = (O_0, \vec{X}_0, \vec{Y}_0, \vec{Z}_0)$ which defines the input shaft position. In fact, only $O_0$, the shaft starting point, and $\vec{Z}_0$, the shaft orientation, are related to the practical problem. $\vec{X}_0$ and $\vec{Y}_0$ are just required for the calculations
- a coordinate system $R_S = (O_S, \vec{X}_S, \vec{Y}_S, \vec{Z}_S)$ which defines the output shaft position : $O_S$ is the shaft terminal point and $\vec{Z}_S$ the axis orientation
- the volume within which all the reducer components must be contained, the most simple case being a parallelepiped defined by two opposite points $P_m$ and $P_M$

- the objective function the geometrical structure has to optimize. Hereafter it is considered that the overall dimensions of the reducer must be minimal

Geometrical specifications are illustrated in Fig. 4.

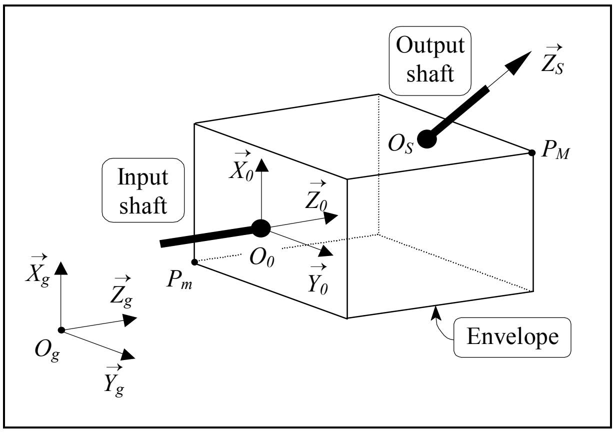



**Fig. 4.** Geometrical specifications

## 4 . Towards a closed chain synthesis problem

To illustrate our purpose in this section, the example of a three stage *3D.S.R.* made of two successive bevel gearing stages, followed by a cylindrical gearing, as shown in Fig. 5a, is considered.

The first level dimensional synthesis problem to be solved consists in determining all the intermediate shaft spatial positions and all the stage general dimensions (i.e. those presented in stage skeletons). The nature of this problem is not the one traditionally encountered in the field of kinematics, as the motion transformation law is not the aim of our study. Each stage is homocinetic, so the overall linkage is also homocinetic.

All input, intermediate and output shafts are linked by a rotational joint to the housing, so their axes are fixed in space and there is no other motion than the rotations about these axes. The problem under study is not influenced by these rotations, which may be ignored. In this case, all shafts may be considered as rigid bodies completely linked to the housing. Moreover, each stage may be replaced by its associated skeleton. This constitutes a first transformation of the problem leading to the overall skeleton of the *3D.S.R.* illustrated in Fig. 5b.

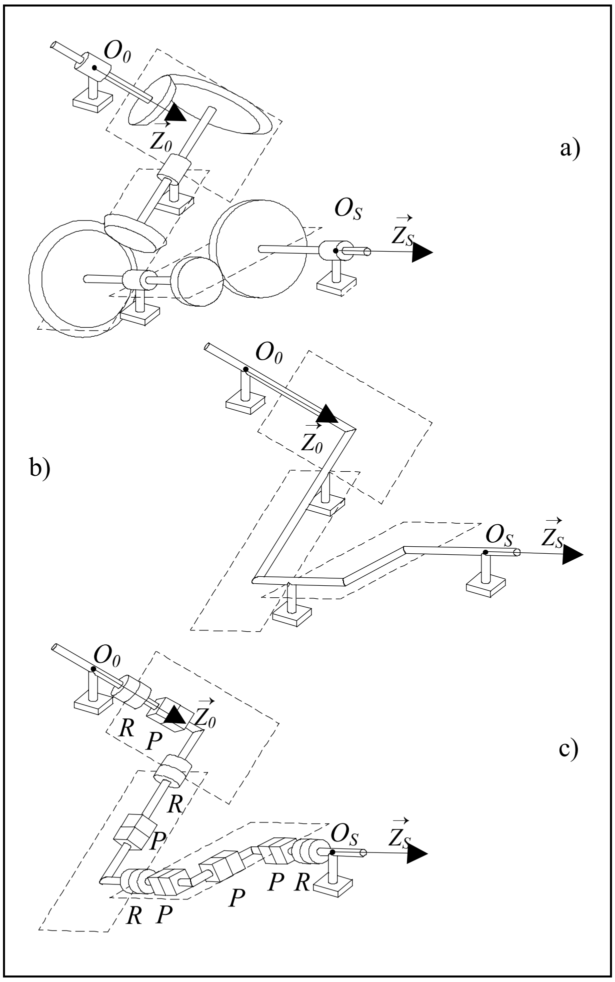



**Fig. 5.** Problem transformations

In order to search for the best positions of all intermediate shafts and the best general dimensions of all stages, some structural constraints may be artificially relaxed. All intermediate shafts are liberated from the housing and joints are added in such a way that the search becomes possible :

- a prismatic joint *P* is introduced wherever a variable length appears in the skeleton. It gives the possibility of simulating modifications in the reducer geometry by extension or compression of the associated dimension. A final prismatic joint is added onto the output axis to model the output shaft length which has not been represented within any skeleton
- a rotational joint *R* is introduced at each extremity of the *3D.S.R.* and between two successive stages. It represents the possibility of modifying the reducer geometry by rotating one stage with respect to the previous one around their common linking shaft.

In such a way, the relative position of the different stage elements is preserved, so the geometrical architecture of each stage is not affected by the transformation. For instance, all angles $\varphi$ remain unchanged. Figure 5c illustrates this second and last transformation of the problem. The initial problem has become the *RPRPRPPPR* study, which belongs to the well-known class of closed chain synthesis problems.

## 5 . The mDH notations

The system obtained from the transformation presented above is considered as being composed of *NJ* joints and *(NJ+1)* links, links *(0)* and *(NJ)* belonging to the same fixed base. The "modified Denavit-Hartenberg's" notations (mDH) proposed by Khalil and Kleinfinger [12] are such that :

- Joint $J_i$ connects links *(i-1)* and link *(i)*
- A coordinate frame $R_i$ is assigned fixed with respect to link *(i)*
- The axis of joint $J_i$ is supposed to lie along $\vec{Z}_i$
- The $\vec{X}_{i-1}$ axis is defined as $\vec{X}_{i-1} = \vec{Z}_{i-1} \wedge \vec{Z}_i$ .

Four parameters define the relative positions of two successive coordinate systems (see Fig. 6) :

- $\alpha_i$ angle between $\vec{Z}_{i-1}$ and $\vec{Z}_i$ about $\vec{X}_{i-1}$
- $d_i$ distance between $O_{i-1}$ and $\Omega_i$
- $\theta_i$ angle between $\vec{X}_{i-1}$ and $\vec{X}_i$ about $\vec{Z}_i$
- $r_i$ distance between $\Omega_i$ and $O_i$ .

The variable associated with joint $J_i$ , denoted by $q_i$ , is $\theta_i$ if $J_i$ is rotational or $r_i$ if $J_i$ is prismatic.

Hence :
$$q_i = \theta_i\left(1-\sigma_i\right)+r_i\sigma_i \qquad (1)$$

where $\sigma_i = 0$ if $J_i$ is rotational and $\sigma_i = 1$ if $J_i$ is prismatic.

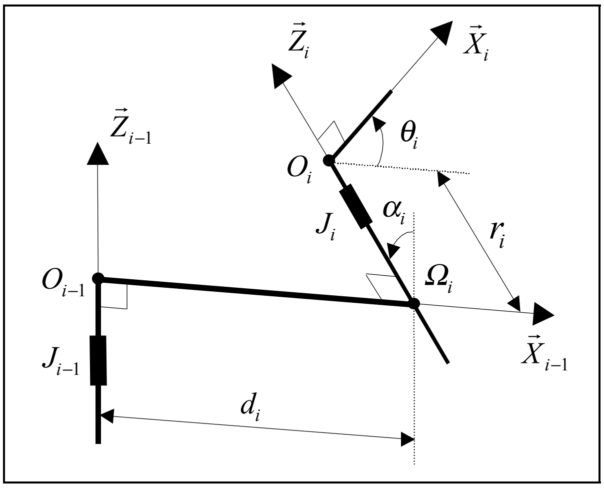



**Fig. 6.** mDH notations

The transformation matrix which defines the frame $R_i$ with respect to frame $R_{i-1}$ is equal to :

$$[T]_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & d_i \\ \cos\alpha_i \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i & -r_i \sin\alpha_i \\ \sin\alpha_i \sin\theta_i & \sin\alpha_i \cos\theta_i & \cos\alpha_i & r_i \cos\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

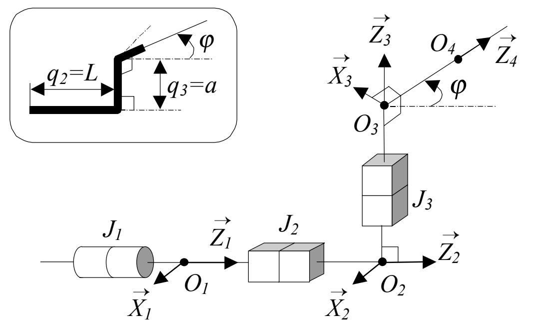
And simple-closed chains are such that :

$$[T]_0^{NJ} = [T]_0^1 . [T]_1^2 .... [T]_{NJ-1}^{NJ} \quad (3)$$

where the right member represents the successive changes among the links connected by joints and the left member is the closure transformation between coordinate systems $R_0$ and $R_{NJ}$ defined in the same fixed base.

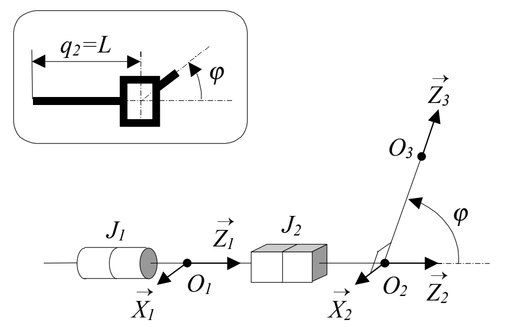
## 6 . Elementary geometrical models according to stage categories

The aim being to build a tool based on standard unit manipulation, a pre-definite geometrical model is associated with each stage category. This definition is done in accordance with the principles of section 4. A prismatic joint takes the place of each variable length. A rotational joint is added at the beginning of the chain. Figures 7 and 8 give the two elementary geometrical models, each one assigned to the respective stage category. The last vector prefigures the first link direction of the next joint. Tables of parameters are drawn up using the mDH notations.




| | $J_1$ | $J_2$ | $J_3$ | $J_4$ |
|---|---|---|---|---|
| $\sigma_i$ | 0 | 1 | 1 | |
| $\alpha_i$ | | 0 | $\pi/2$ | $\pi/2$ |
| $d_i$ | | 0 | 0 | 0 |
| $\theta_i$ | $q_1$ | 0 | $\pi+\varphi$ | |
| $r_i$ | 0 | $q_2$ | $q_3$ | |

**Fig. 7.** Geometrical model for non concurrent shaft stages




| | $J_1$ | $J_2$ | $J_3$ |
|---|---|---|---|
| $\sigma_i$ | 0 | 1 | |
| $\alpha_i$ | | 0 | $\varphi$ |
| $d_i$ | | 0 | 0 |
| $\theta_i$ | $q_1$ | 0 | |
| $r_i$ | 0 | $q_2$ | |

**Fig. 8.** Geometrical model for concurrent shaft stages

## 7 . Automatic construction of the geometrical model associated with a *3DSR* problem

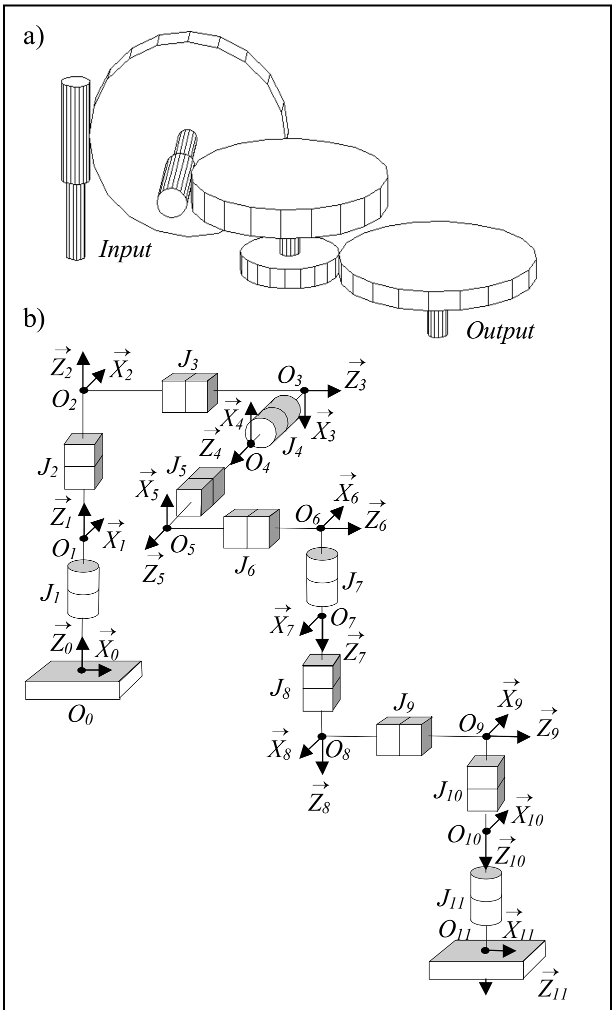
Hence, it is easy to deal with the case of any reducer made of serially connected speed reducing stages. The associated geometrical model may be obtained in this way :

- the first elementary stage table is set and completed with parameters $\alpha_1 = d_1 = 0$ considering that the first joint $J_1$ is a rotational one
- the next stage table is added like a puzzle piece at the end of the previous table and so on up to the last stage
- two columns are added at the end in order to materialize a prismatic joint representing the possible length variation of the output shaft and a rotational joint representing the possible rotation of the structure about the output axis.

Let us consider the example of a three stages *3D.S.R.* whose constitution is :

- two successive worm gearing stages with $\varphi = 90°$
- an external/external cylindrical gearing stage with shafts on opposite sides $(\varphi = 0°)$.

Figure 9a shows the diagram of the reducer given in an arbitrary starting position. Figure 9b illustrates the whole geometrical model which can be automatically generated and Fig. 9c the associated parameter table.




c)

| | *First stage* | | | *Second stage* | | | *Third stage* | | | *Output shaft* | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ | $J_8$ | $J_9$ | $J_{10}$ | $J_{11}$ |
| $\sigma_i$ | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| $\alpha_i$ | 0 | 0 | $\pi/2$ | $\pi/2$ | 0 | $\pi/2$ | $\pi/2$ | 0 | $\pi/2$ | $\pi/2$ | 0 |
| $d_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\theta_i$ | $q_1$ | 0 | $3\pi/2$ | $q_4$ | 0 | $3\pi/2$ | $q_7$ | 0 | $\pi$ | 0 | $q_{11}$ |
| $r_i$ | 0 | $q_2$ | $q_3$ | 0 | $q_5$ | $q_6$ | 0 | $q_8$ | $q_9$ | $q_{10}$ | 0 |

**Fig. 9.** *3D.S.R* example and its whole geometrical model