APPLICATION OF A FUZZY LOGIC ORDERING METHOD TO PRELIMINARY MECHANISM DESIGN

APPLICATION D'UNE MÉTHODE DE CLASSEMENT PAR LA LOGIQUE FLOUE À LA CONCEPTION PRELIMINAIRE DE MÉCANISMES

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ABSTRACT

A fuzzy logic method was implemented and applied to preliminary mechanism design. This method permits to order a set of previously automatically designed mechanisms and fits perfectly to the vagueness of the designer knowledge, at least at this early stage of design. The comparison algorithm between two mechanisms, which is based on Mamdani implications, is detailed with an example. In spite of its computational cost, this method offers an interesting alternative to conventional multi-criteria methods because it directly uses qualitative data.

RÉSUMÉ

Nous proposons d’appliquer une méthode de logique floue à la conception préliminaire de mécanismes. Cette méthode nous permet de classer des mécanismes, conçus préalablement par une méthode automatique, et s’adapte parfaitement au caractère vague des connaissances du concepteur à ce stade précoce de la conception. L’algorithme de comparaison entre deux mécanismes, qui est basé sur les implications de Mamdani, est présenté en détails à travers un exemple. Malgré son coût en temps de calcul, cette méthode offre une alternative intéressante aux méthodes multi-critères conventionnelles car elle utilise directement des grandeurs qualitatives.

I . INTRODUCTION

Fuzzy logic knew a considerable growth in various domains, even if it is still underused in mechanical engineering. However, it seems to be of promising interest in the early stages of design, where qualitative reasoning is mostly performed, without taking exact dimensions into account. In order to evaluate the advantages of fuzzy logic for this purpose, it was integrated in a geared mechanism preliminary design system. This system already ran correctly with a classical multi-criterion ordering method. We wondered if fuzzy logic was of any interest for this example in order to improve some points for which the classical method was not totally satisfactory.

II . NATURE OF THE PROBLEM

We would like to improve our existing geared mechanism preliminary design system [FAU.97] with fuzzy logic. This program provides a list of mechanical solutions which comply given specifications. It is based on a three step method (Fig. 1) :

1. Scanning of the mechanical solution domain.
2. Elimination of the candidates which do not meet the design rules.
3. Sorting of the remaining candidates by preference order

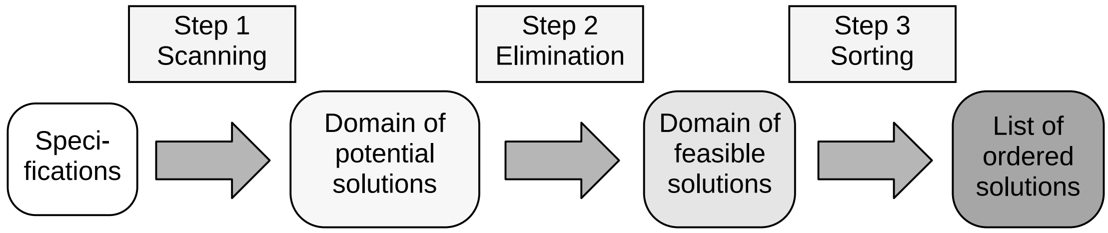


Fig.1 : Structure of the preliminary mechanism design program
Structure du programme de conception préliminaire de mécanismes

From now on, we focus on step three. Each of the mechanical solutions is given a mark, which will permit the subsequent ordering. The following hypothesis are given :

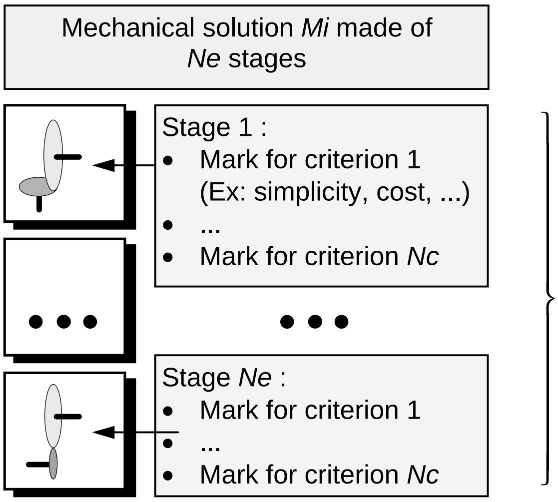
- Each mechanical solution *Mi* is made of *Ne* stages, elementary mechanisms serially connected and taken from a mechanism database.

- Each of the elementary mechanisms was previously given several marks by an expert, according to *Nc* criteria. The marks are recorded in the database.

The followed process for calculating the mark of a mechanical solution can be found below (Fig. 2) :

- The mark of solution *Mi* is the mean of the *Ne* elementary stage marks

- Weighting coefficients *Kc* permit to adjust the respective influence of criteria.



Mark of mechanical solution *Mi*

$$= \frac{\sum_{c=1}^{Nc} Kc \cdot \left[\frac{\sum_{s=1}^{Ns} \left(\begin{array}{l} \text{Mark of Stage } s \\ \text{for Criterion } c \end{array} \right)}{Ns} \right]}{\sum_{c=1}^{Nc} Kc}$$

Fig.2 : “Traditional” way of obtaining the mark for a mechanical solution Mi
Calcul “traditionnel” de la note d’un mécanisme-solution Mi

<u>Advantages of this rather “traditional” multi-criteria process :</u>

- Simplicity (for programmer as well as for user)
- Execution speed
- Reliable and practically confirmed results.

<u>Drawbacks :</u>

- Designers generally think “qualitative” with linguistic rather than numerical variables. They know, for instance, that a cylindrical gear is “less expensive” and “easier to mount” than a conical one. By using such a method, they should give a mark (out of 100 for example) to each elementary mechanism in the database (Tab. 3).

	Gear type	
Nature of criterion	Cylindrical	Conical
Manufacturing cost (mark out of 100)	50	15
Mounting simplicity (mark out of 100)	50	10

Tab.3 : Examples of marks given to mechanisms
Exemples de notes affectées aux mécanismes

It is difficult to justify every single point in the marks : they rather have a relative meaning. If conical gears are given a mark of 15 instead of 40, it is because several other mechanisms must be inserted between cylindrical and conical gears. Moreover, these marks must be adjusted and spread out each time we add or suppress elementary mechanisms from the database.

- Obviously, the global mark is a mixing of several quantities which are not of the same nature.

Consequently, these reservations justify our interest for a new type of process based on fuzzy logic. We will try to determine if it is worth using fuzzy logic and what are its strong and weak points in a mechanical context like ours.

III . FUZZY ALGORITHM

We now introduce, through a simple example, the fuzzy algorithm which was chosen for comparing two solutions. As soon as the “comparison” notion is defined, it will be easy to implement a “sorting” method for step 3.

1 . Simple example

Two mechanical solutions, named A and B, each one made of two stages (respectively A1, A2 et B1, B2) and evaluated according to criteria (named C1 and C2) must be compared (Tab. 4).

	Solution A		Solution B	
Criteria	Stage A1	Stage A2	Stage B1	Stage B2
Criterion C1	G	VB	G	M
Criterion C2	VB	B	B	VG

With the following abbreviations :

VB : Very Bad	B : Bad	M : Medium	G : Good	VG : Very Good

Tab.4 : Nature of the compound mechanisms to be compared
Nature des mécanismes composés à comparer

These initial data are of qualitative nature and relatively vague. They are expressed in a way close to the one an expert uses to give his point of view.

In this simple case, five quality classes are used, each one being defined by a triangular shape membership function (Fig. 5). Such a distribution between two extremes, with a medium and two intermediate classes will suit small examples like this one but might be refined subsequently.

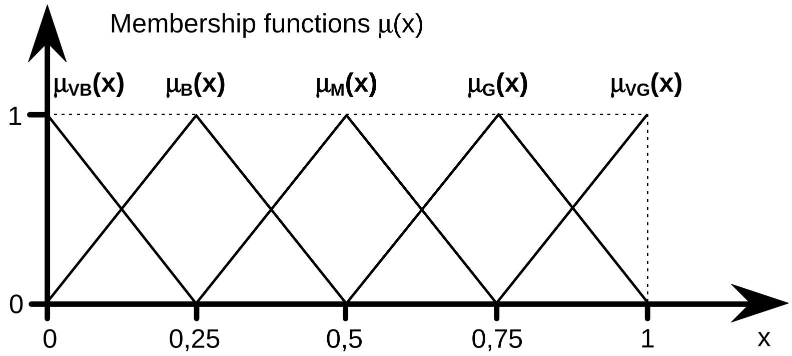


Fig.5 : Membership functions of the five quality classes
Fonctions d’appartenance des 5 classes de qualité

2 . Comparison principle

Comparison between two solutions A and B is a three phase process (Fig.6) :

- First each solution is evaluated according to each criterion ;
- Then solutions are compared criterion by criterion ;
- Finally, all the comparisons are mixed up in a global comparison.

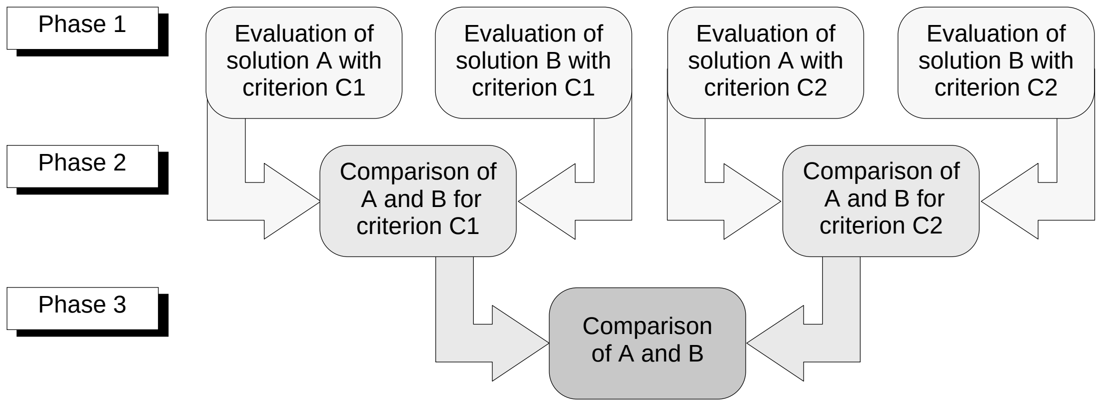


Fig.6 : Comparison principle between two solutions A and B
Principe de la comparaison entre deux solutions A et B

3 . First step : Evaluation of each solution according to a criterion

For instance, let us evaluate quality of the constitutive stages of solution A according to criterion C1 (Tab.7).

	Solution A	
Criteria	Stage A1	Stage A2
Criterion C1	G	VB

Tab.7 : Evaluation of solution A according to criterion C1
Evaluation de la solution A selon le critère C1

According to Mamdani method, the barycentre of the areas delimited by the corresponding membership functions should be calculated (Fig. 8).

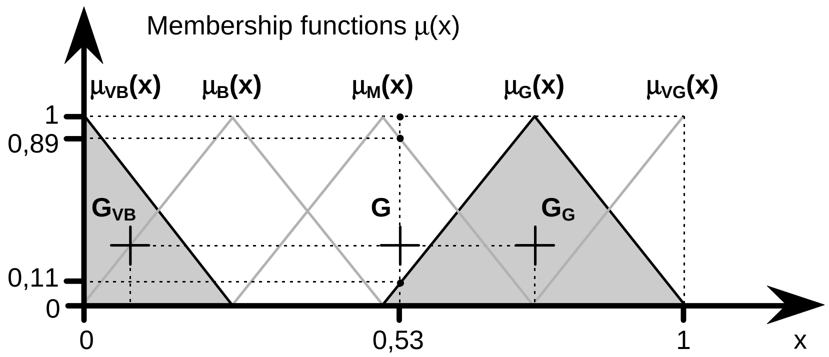


Fig.8 : Mamdani barycentre principle
Principe du barycentre de Mamdani

Then, we get the abscissa of the global barycentre (*x = 0,53*) which gives the membership degrees of solution A to each of the five quality classes (Tab. 9).

VB	B	M	G	VG
0,00	0,00	0,89	0,11	0,00

Tab.9 : Membership degrees of solution A to the 5 quality classes according to C1
Degrés d'appartenance de la solution A aux 5 classes de qualité selon C1

This calculation is repeated identically for solution B and criterion C2. The corresponding results take place in Table 10 :

Evaluation of :	Value of x	VB	B	M	G	VG
A according to C1	0,53	0,00	0,00	0,89	0,11	0,00
B according to C1	0,63	0,00	0,00	0,50	0,50	0,00
A according to C2	0,19	0,22	0,78	0,00	0,00	0,00
B according to C2	0,47	0,00	0,11	0,89	0,00	0,00

Tab.10 : Membership degrees to the 5 quality classes
Degrés d'appartenance aux 5 classes de qualité

4 . Second step : Comparison of the solutions for each criterion

The problem is now to compare solution B to solution A, knowing their quality degrees according to criterion Ci. The following rules are given :

- **If** (B has the same quality degree as A) **Then** (B is Equivalent to A)
- **If** (B has one more quality degree than A) **Then** (B is Superior to A)
- **If** (B has two or more quality degrees than A) **Then** (B is Very Superior to A)
- ...

These rules are summarized in Table 11, where the five possible degrees of comparison can be seen.

	Solution B				
Solution A	VB	B	M	G	VG
VB	E	S	VS	VS	VS
B	I	E	S	VS	VS
M	VI	I	E	S	VS
G	VI	VI	I	E	S
VG	VI	VI	VI	I	E

With the following abbreviations :

VI : Very Inferior	I : Inferior	E : Equivalent	S : Superior	VS : Very Superior

Tab.11 : Comparison of B with A according to criterion Ci
Comparaison de B par rapport à A selon un critère Ci

A and B may now be compared according to C1. The previous rule :

If (B has the same quality degree as A) **Then** (B is Equivalent to A)

can be reformulated in the following way :

If ((B='VB' **And** A='VB') **Or**
(B='B' **And** A='B') **Or**
(B='M' **And** A='M') **Or**
(B='G' **And** A='G') **Or**
(B='VG' **And** A='VG')) **Then** (B is Equivalent to A)

We then use Mamdani definition [FOU.94] for the **And** connector :

$$\mu_{P \text{ and } Q}(x, y) = \min(\mu_P(x), \mu_Q(y))$$

This definition is applied and illustrated in Table 12 : the numerical value in each cell is obtained by taking the minimum of the corresponding row and column values.

	Solution B (with membership values)				
Solution A (val.)	VB (0,00)	B (0,00)	M (0,50)	G (0,50)	VG (0,00)
VB (0,00)	E (0,00)	S (0,00)	VS (0,00)	VS (0,00)	VS (0,00)
B (0,00)	I (0,00)	E (0,00)	S (0,00)	VS (0,00)	VS (0,00)
M (0,89)	VI (0,00)	I (0,00)	E (0,50)	S (0,50)	VS (0,00)
G (0,11)	VI (0,00)	VI (0,00)	I (0,11)	E (0,11)	S (0,00)
VG (0,00)	VI (0,00)	VI (0,00)	VI (0,00)	I (0,00)	E (0,00)

Tab.12 : Comparison of B with A according to C1 (example)
Comparaison de B par rapport à A selon C1 (exemple)

Then, Mamdani definition for the **Or** connector is used :

$$\mu_{P \text{ or } Q}(x, y) = \max(\mu_P(x), \mu_Q(y))$$

Next, Table 13 is constructed in this way : for each comparison degree in Table 12 (grey cell areas), the only value to be kept is the maximum one.

Criteria :	VI	I	E	S	VS
Criterion C1	0,00	0.11	0,50	0,50	0,00
Criterion C2	0,00	0,00	0,11	0,78	0,22

Tab.13 : Degrees of comparison of B with A
Degrés de comparaison de B par rapport à A

Thus, for instance, we can see from Table 13 that there is a 50 % probability that B is superior to A according to C1 and 78 % according to C2.

5 . Third step : Final comparison

Our purpose is now to combine the various comparisons (one per criterion) in a unique one. We use the following reasoning : “If there is superiority according to C1 and inferiority according to C2 then there is equivalence”. This can be translated into :

If ((C1=‘S’) **And** (C2=‘I’)) **Then** (‘E’)

Next, we apply the same process as in Step 2 (min operator for Table 14, max operator for Table 15).

Table 14	Criterion C2 (with membership values)				
Criterion C1 (val.)	VI (0,00)	I (0,00)	E (0,11)	S (0,78)	VS (0,22)
VI (0,00)	I (0,00)	I (0,00)	I (0,00)	I (0,00)	E (0,00)
I (0,11)	I (0,00)	I (0,00)	I (0,11)	E (0,11)	S (0,11)
E (0,50)	I (0,00)	I (0,00)	E (0,11)	S (0,50)	S (0,22)
S (0,50)	I (0,00)	E (0,00)	S (0,11)	S (0,50)	S (0,22)
VS (0,00)	E (0,00)	S (0,00)	S (0,00)	S (0,00)	S (0,00)

Tab.14 : Mixing criteria C1 and C2
Mixage des critères C1 et C2

I	E	S
0,11	0,11	0,50

Tab.15 : Final comparison of B with A
Comparaison finale de B par rapport à A

Proposition "S" (i.e. B > A) seems to be the most plausible because its membership function has the highest value of all. Thus, this result confirms what Table 4 already seemed to suggest.

IV . COMPARATIVE EXAMPLE

Now, let us compare the traditional ordering method with its fuzzy version. As we did not want to loose the subtleties contained in the expert database, we chose to use a seven quality degree representation (instead of five for the example). Of course, this choice leads to a heavier computational load.

For a same set of 1016 mechanical solutions, the ordering time is about three minutes long for the fuzzy algorithm instead of about ten seconds for the traditional one.

As for the quality of the resulting set (that is to say the good ordering of solutions), it is similar in both cases.

V . CONCLUSION

Undeniably, fuzzy logic suits perfectly the representation of qualitative information, such as those used at the beginning of a design process. The fuzzy algorithm previously described gives correct results, similar to those obtained with the traditional method. It is more satisfactory for representing expert knowledge and fits well to ordinary human expression.

However, these qualities are obtained at the cost of heavier computations.

References

[DUB.94] DUBOIS Didier, PRADE Henri. « *Ensembles flous et théorie des possibilités : notions de base* », ARAGO 14 - Logique Floue, Masson (France), 1994, N° ISBN 2-22-584474-7, pp. 29-62.

[FAU.97] FAUROUX Jean-Christophe, SARTOR Marc. « *Conception qualitative de mécanismes. Application aux réducteurs à engrenages* », AUM'97, Poitiers (France), 1er-5 septembre 1997, Vol. 3, pp. 319-322.

[FOU.94] FOULLOY Laurent, TITLI André. « *Aspects généraux de la commande floue* », ARAGO 14 - Logique Floue, Masson (France), 1994, N° ISBN 2-22-584474-7, pp. 64-79.

[THU.92] THURSTON D.L., CARNAHAN J.V. « *Fuzzy ratings and utility analysis in preliminary design evaluation of multiple attributes* », Journal of Mechanical Design, Vol. 114, Dec. 1992, pp. 648-658.